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Thermal radiation on oscillatory flow past a moving vertical plate in a time varying gravity field

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Abstract

Thermal radiation on oscillatory flow of a viscous incompressible fluid of an optically dense medium past a moving hot vertical plate under the influence of a time varying gravity field by using Rosseland approximation is investigated. Since the flow is characterised by a time varying gravity field, a g-jitter force is associated with microgravity field in space. In a g-jitter driven flow, the empty space above the leading edge of the medium corresponds to a crystal growth in space; the net mass flow rate in the system is zero. In a time varying gravity field, the flow is characterised by the oscillating velocity near the plate surface with reference to a phase angle and to increase the fluid velocity with increase in phase angle due to impulsive onset into motion. The effects of radiation parameter, Prandtl number and Grashof number on the velocity field are analysed. The effect of Nusselt number at the plate is discussed.

Keywords: Thermal radiation; G-jitter force; Oscillatory flow; Grashof number; Rosseland approximation; Time varying gravity.

1. Introduction

In recent year, authors have been studied the concept of a controlled thermonuclear fusion reaction of the Sun in the presence of a magnetic mirror subject to a traveling magnetic field to deal with resonance with a decisive importance to a forced oscillation. Mention may be made of their works of Ghosh [1], Ghosh et.al [2-3] and Ghosh [4]. The present study deals with the hydrodynamic flow of a time varying gravity field with reference to an optically thick medium. The Sun is an exposure of a resonance to exerts its influence of a time varying gravity field with a decisive importance to a differential rotation of the Sun when a forced oscillation is taken into account. In a time varying gravity field which oscillates harmonically with time, a gravitational pull in a vacuum is attracted by the Sun and the stars and planets are in motion in a space time interval. In a time varying gravity field, the importance of a microgravity field in space lies in its radiation effect with reference to an electromagnetic radiation from the Sun. A time varying gravity field inside the solar atmosphere reveals to an oceanic circulation that effects the change of climatic condition. Although, a controlled thermonuclear fusion reaction of the Sun communicates a radio wave to become a swollen of oceanic circulation and an environmental change leads to a depression on oceanic circulation due to a differential rotation of the sun. In a time varying gravity field inside the Sun, the force of attraction reveals to the strength of gravity that may vary from one planet to the other. In such a situation, gravitational force is attracted by the planet 'Mars' to exhibit ultraviolet radiation and some of the stars are visible in the sky of 'Mars' due to the existance of ultraviolet radiation. The influence of gravity in the planet 'Mars' appears to be a change of environment with the formation of water from the ice zone of the 'Mars'. It is expected that the source of water in the planet 'Mars' becomes significant. It is important to note that the origin of species can be found in the planet 'Mars' with reference to a controlled thermonuclear fusion reaction of Sun under the inflence of a magnetic mirror. In a time varying gravity, buoyancy force exerted by the convective part of the surface of the Sun. In general, free and forced convective flow has been studied by Mazumder et.al [5], Gupta [6], Ghosh and Bhattacharjee [7] and Pop et.al [8].

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In relevance to the physical situation of interest, the aim of a present investigation is to study of an extension of Stoke’s flow exerted by the time varying gravity field by employing Rosseland approximation. It lies in its application of designing a space craft propulsion system. Since time varying gravity field is associated with microgravity field in space, a g-jitter driven flow in a free space communicates in crystal growth in space; the net mass flow rate in the system is zero. It exerts its influence of space fluid system design and interpreting the experimental measurement in microgravity flow and heat transfer system. Nevertheless, thermal radiation of an optically thick fluid has been carried out by England and Emery [9], Naroua et.al [10], Raptis and Perdakis [11], Muthucumaraswamy and Ganesan [12], Davis [13], Bestman [14], Compo and Schuler [15] and Ghosh et.al [16] with different conditions and configurations. In a realistic situation, the present problem deals with a new investigation with reference to a time varying gravity field by using Rosseland approximation for an optically dense medium. An attempt has been made to focuss the Stoke’s flow of a viscous incompressible fluid past a vertical flat plate driven by a time varying gravity field with a decisive importance to a microgravity field in an optically dense medium just as with molecular conductivity the transfer of radiant energy in a medium can be compared with diffusion transfer. Here, the effect of interphoton collision is predominant.

2. Formulation of the problem and its solutions

Consider an unsteady flow of a viscous incompressible fluid occupying a semi-infinite region of space bounded by an infinite hot vertical plate. The flow is induced by a time varying gravity field which oscillates harmonically with time. The time varying gravity field will generate an oscillatory free convection velocity field. It is considered that the velocity of the fluid far away from the plate is zero. The flow is considered in an optically thick gray gas with a decisive importance to free convection and radiation. We choose the co-ordinate system in such a way that the x' -axis is taken along the plate and y' -axis is normal to it. It is considered that all the fluid properties are constant except the influence of density variation on the body force term. The radiative heat flux in the x' -direction is considered negligible in comparison to the y' direction.

Under Boussinesq approximation, the momentum equation becomes

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g(t') \beta (T' - T'_\infty) \quad (1)$$

The equation of continuity becomes

$$\nabla \cdot u' = 0 \quad (2)$$

The equation of energy transfer is

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y'} \quad (3)$$

Where $u', t', \nu, \rho, k, C_p, g(t'), \beta, T', T'_\infty, q_r$ are respectively, the velocity component along the plate, the time, the kinematic viscosity, the fluid density, the thermal conductivity, the specific heat at constant pressure, the time varying gravity, the coefficient of thermal expansion, the temperature of the fluid, the temperature of fluid far away from the plate and the radiative heat flux.

Since the flow is induced by a time varying gravity field which oscillates harmonically with time, it is reasonably assumed as

$$g(t') = g_0 \cos \omega' t' \quad (4)$$

Where g_0 is the gravitational acceleration and ω' is the frequency

In a time varying gravity field, the flow will generate an oscillatory free convection flow.

The initial and the boundary conditions for velocity and temperature distributions are

$$\begin{aligned} u' = 0, T' = T'_\infty & \text{ for all } y' \geq 0 \text{ and } t' \leq 0, \\ u' = U, T' = T'_w & \text{ at } y' = 0 \text{ for } t' > 0, \\ u' \rightarrow 0, T' \rightarrow T'_\infty & \text{ as } y' \rightarrow \infty \text{ for } t' > 0 \end{aligned} \quad (5)$$

Where T'_w is the temperature at the plate and U is the velocity of the plate.

The radiative heat flux is addressed using the approach outlined by Isachenko et.al [17]. The Rosseland diffusion flux approximation is therefore used leading to a Fourier-type gradient function, viz:

$$q_r = - \frac{4\sigma^*}{3k^*} \frac{\partial T'^4}{\partial y'} \quad (6)$$

Where k^* is the spectral mean absorption coefficient of the medium and σ^* is the Stefan-Boltzman constant.

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be regarded as a linear function of temperature. It can be established by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher order terms. Therefore, T'^4 can be expressed in the following form

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \tag{7}$$

Introducing dimensionless quantities

$$u = \frac{u'}{U}, \quad y = \frac{y'U}{\nu}, \quad \tau = \frac{t'U^2}{\nu}, \quad \omega = \frac{\omega'\nu}{U^2}$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad Pr = \frac{\rho \nu C_p}{k} \text{ and } Gr = \frac{\nu \beta g_0 (T'_w - T'_\infty)}{U^3} \tag{8}$$

Where Pr , g_0 , Gr and T are, respectively, the Prandtl number, the gravitational acceleration, Grashof number and temperature.

Making use of Equations (6) - (7), the energy Equation (3) can be written in dimensionless form subject to Equation (8) such as

$$(1+k_1) \frac{\partial^2 T}{\partial y^2} - Pr \frac{\partial T}{\partial \tau} = 0 \tag{9}$$

Where $k_1 = \frac{16\sigma^* T'^3_\infty}{3k^*k}$ is the radiation parameter.

Using Equation (8), Equation (1) transforms into a dimensionless form such as

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial y^2} + TG_r \cos \omega \tau \tag{10}$$

The corresponding initial and boundary conditions for velocity and temperature field are

$$u = 0, \quad T = 0 \quad \text{for all } y \geq 0 \text{ and } \tau \leq 0,$$

$$u = F(\tau), \quad T = 1 \quad \text{at } y = 0 \text{ for } \tau > 0, \tag{11}$$

$$u \rightarrow 0, \quad T \rightarrow 0 \quad \text{as } y \rightarrow \infty \text{ for } \tau > 0$$

where $F(\tau) = \cos \omega \tau$

By applying Laplace transform of $F(\tau)$ it turns into

$$L\{F(\tau)\} = \frac{s}{s^2 + \omega^2} = \frac{1}{2} \left[\frac{1}{s+i\omega} + \frac{1}{s-i\omega} \right] \tag{12}$$

By employing Laplace transform technique, Equations (10) and (9) become

$$su^* = \frac{\partial^2 u^*}{\partial y^2} + T^* Gr \frac{s}{s^2 + \omega^2} \tag{13}$$

$$(1+k_1) \frac{\partial^2 T^*}{\partial y^2} - Pr s T^* = 0 \tag{14}$$

The corresponding initial and boundary conditions for velocity and temperature field are

$$u^* = 0, \quad T^* = 0 \quad \text{for all } y \geq 0 \text{ and } \tau \leq 0$$

$$u^* = \frac{s}{s^2 + \omega^2}, \quad T^* = \frac{1}{s} \quad \text{at } y = 0 \text{ and } \tau > 0 \tag{15}$$

$$u^* \rightarrow 0, \quad T^* \rightarrow 0 \quad \text{as } y \rightarrow \infty \text{ for } \tau > 0$$

Solutions for the velocity distribution and temperature distribution can be obtained by employing Inverse Laplace transform technique subject to Equations (13) and (14) together with the boundary conditions (15) become

$$\begin{aligned}
 u(y, \tau) = & \frac{1}{\omega^2} D_1 G_r \left[\operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} \right) - \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{c_1}{\tau}} \right) \right] + \\
 & \left(\frac{1}{4} - \frac{1}{4\omega^2} D_1 G_r \right) \left[e^{i\omega\tau} \left\{ e^{y\sqrt{i\omega}} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} + \sqrt{i\omega\tau} \right) + e^{-y\sqrt{i\omega}} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} - \sqrt{i\omega\tau} \right) \right\} + \right. \\
 & \left. e^{-i\omega\tau} \left\{ e^{y\sqrt{-i\omega}} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} + \sqrt{-i\omega\tau} \right) + e^{-y\sqrt{-i\omega}} \operatorname{erfc} \left(\frac{y}{2\sqrt{\tau}} - \sqrt{-i\omega\tau} \right) \right\} \right] + \\
 & \frac{1}{4\omega^2} D_1 G_r \left[e^{i\omega\tau} \left\{ e^{y\sqrt{i\omega c_1}} \operatorname{erfc} \left(\frac{y\sqrt{c_1}}{2\sqrt{\tau}} + \sqrt{i\omega\tau} \right) + e^{-y\sqrt{i\omega c_1}} \operatorname{erfc} \left(\frac{y\sqrt{c_1}}{2\sqrt{\tau}} - \sqrt{i\omega\tau} \right) \right\} + \right. \\
 & \left. e^{-i\omega\tau} \left\{ e^{y\sqrt{-i\omega c_1}} \operatorname{erfc} \left(\frac{y\sqrt{c_1}}{2\sqrt{\tau}} + \sqrt{-i\omega\tau} \right) + e^{-y\sqrt{-i\omega c_1}} \operatorname{erfc} \left(\frac{y\sqrt{c_1}}{2\sqrt{\tau}} - \sqrt{-i\omega\tau} \right) \right\} \right] \tag{16}
 \end{aligned}$$

$$T(y, \tau) = \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{c_1}{\tau}} \right) \tag{17}$$

Where $c_1 = \frac{Pr}{1+k_1}$ and $D_1 = \frac{1}{c_1 - 1}$

Nusselt number at the plate $y=0$ can be obtained from (17)

$$-\frac{\partial T}{\partial y} \Big|_{y=0} = \frac{1}{\sqrt{\pi\tau}} \sqrt{\frac{Pr}{1+k_1}} \tag{18}$$

3. Results and discussion

An interpretation of the physical insight into the flow regime exerted by the numerical results for the velocity distributions are depicted graphically in Figs. 1 to 4 versus y for various values of k_1 , τ , Pr , ω and $\omega\tau$.

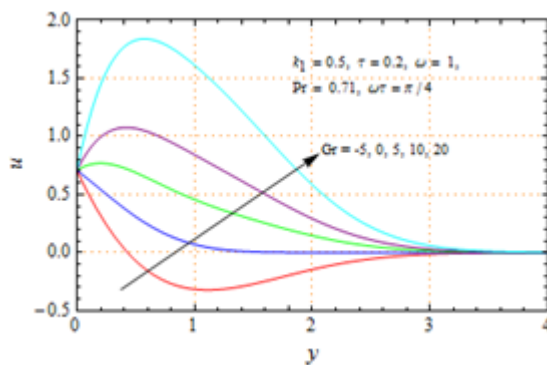


Figure 1 Velocity distributions for Gr

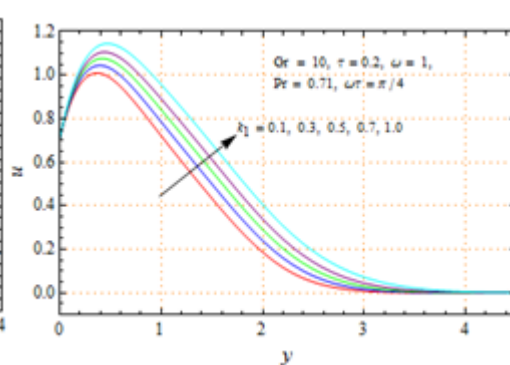


Figure 2 Velocity distributions for k_1

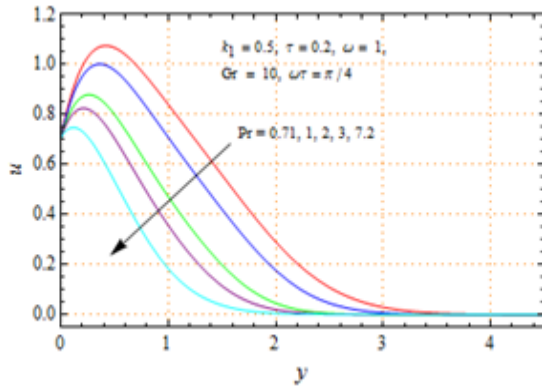


Figure 3 Velocity distributions for Pr

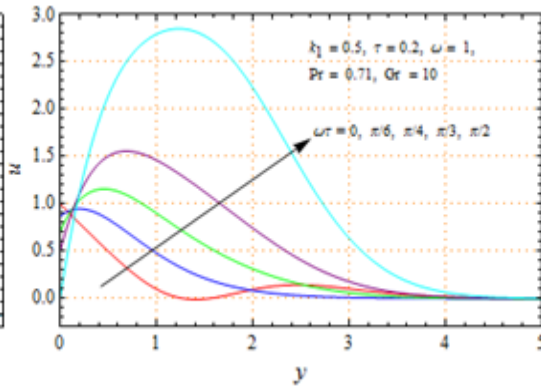


Figure 4 Velocity distributions for $\omega\tau$

It is evident from Figs. 1 to 4 that the profiles are skewed in nature and ultimately converge near the plate under the influence of a Rosseland approximation. The skewness is characterised by the buoyancy force in a time varying gravity field. Fig.1 shows that the velocity increases with increase in buoyancy force (G_r). The thermal Grashof number becomes a significant behaviour with the relative effect of thermal buoyancy (due to density variation) force to the viscous force in the flow regime so that the positive value of G_r corresponds to cooling of plate with reference to natural convection. It is observed that in a free convection effect, the transient velocity accelerates with the increase in thermal buoyancy force. The fluid velocity increases due to increase in buoyancy force to raise its peak value near the plate surface and then decreases monotonically to the zero-free stream value hold true for the far field condition. Fig.2 illustrates the influence of the Rosseland radiation- conduction parameter (k_1) on velocity profiles for $P_r=0.71$ ($P_r < 1$), so that heat diffuses faster than momentum in the regime. Due to strong thermal radiation flux, an increase in k_1 corresponds to a slight increment in the velocity in close proximity to the plate surface. There is a linear decay close to the plate converges the velocity profile for any k_1 . Fig.3 demonstrates that with the increase in Prandtl number (P_r), the fluid velocity decreases near the plate surface. In relevance to the physical situation of interest it reveals that the significance of fluids with high Prandtl number determine high viscosity so that fluids move slowly with smaller value of Prandtl number which correspond to increasing thermal conductivity and therefore, heat generates to diffuse away from the heated surface and move rapidly than of higher value of Pr. Fig.4 reveals that the fluid velocity is of oscillatory character near the plate surface on increasing the phase angle ($\omega\tau$) while the fluid velocity increases with an increase in phase angle ($\omega\tau$). This situation reveals that in a time varying gravity field, the fluid velocity increases with increase in $\omega\tau$ with reference to an impulsive onset of the motion. There arises a phase lead on molecular diffusion region with interphoton collision.

Table 1 Nusselt number at the plate for $P_r= 0.71$ (Air)

k_1 τ	0.1	0.3	0.5	0.7	1.0
0.2	1.01354	0.93232	0.86794	0.81529	0.75166
0.4	0.71668	0.65925	0.61373	0.57650	0.53150
0.6	0.58517	0.53827	0.50111	0.47071	0.43397
0.8	0.50677	0.46616	0.43397	0.40764	0.37583
1.0	0.45327	0.41694	0.38816	0.36461	0.33615

The heat transfer from the plate is characterised by the Nusselt number to exhibit the effects of radiation and time on the flow behaviour where the Prandtl number ($P_r=0.71$ for air) is kept fixed. This has been shown from Table 1. It is evident from Table 1 that the Nusselt number at plate ($y=0$) decreases with an increase in either radiation parameter (k_1) or the time (τ). The effect of Prandtl number plays a significant role on diffusion concept of the flow medium. The physical significance of the results obtained from Table 1 that the heat transfer from the plate to the fluid with low Prandtl number has higher thermal conductivity. The higher thermal conductivity means fluid has affinity for heat and so low Prandtl fluid attains comparatively higher temperature. As a result, the Nusselt number at the plate leads to fall the temperature from plate to the fluid.

4. Conclusion

Thermal radiation on oscillatory flow of an optically dense medium past a moving hot vertical plate in a time varying gravity field is investigated. Since a time varying gravity field is associated with microgravity field in space, a g-jitter driven flow the empty space above the leading edge of the plate becomes significant to a crystal growth in space; the net mass flow rate in the system is zero. The present problem deals with a new investigation with reference to a time varying gravity field exerted by an optically dense medium by using Rosseland approximation. In a time varying gravity field, the flow is characterised by an oscillating velocity near the plate surface with a decisive importance to a phase angle so that the fluid velocity increases with an increase in phase angle due to impulsive onset into motion. The diffusion concept of heat transfer by employing Rosseland approximation finds application of space fluid system design and interpreting the experimental measurement of microgravity flow and heat transfer system.

Compliance with ethical standards

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Disclosure of conflict of interest

Author claims that he has no conflict of interest.

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