



(RESEARCH ARTICLE)



## Multivariate gaussian process incorporated predictive model for stream turbine power plant

Prama Debnath<sup>1</sup> and Mithun Ghosh<sup>2,\*</sup>

<sup>1</sup> Department of Computer Engineering, American International University-Bangladesh, Dhaka, Bangladesh.

<sup>2</sup> Department of Systems and Industrial Engineering, University of Arizona, Tucson, Arizona, USA.

Global Journal of Engineering and Technology Advances, 2022, 12(02), 096–105

Publication history: Received on 30 December 2021; revised on 18 August 2022; accepted on 22 August 2022

Article DOI: <https://doi.org/10.30574/gjeta.2022.12.2.0145>

### Abstract

Steam power turbine-based power plant approximately contributes 90% of the total electricity produced in the United States. Mainly steam turbine consists of multiple types of turbine, boiler, attemperator, reheater, etc. Power is produced through the steam with high pressure and temperature that is conducted by the turbines. The dynamics of the power plant are highly nonlinear considering all these elements in the model. We proposed to capture the dynamics of the power plant through Simulink modeling where data are generated, and a subsequent data-driven predictive modeling approach is used to detect the power generation from these turbines by the multivariate Gaussian process (MGP). The modeling approach is considered to predict the power generation from these turbines which can capture the cross-correlations between the turbines. Also, the sensitivity analysis of the input parameters is constructed for each turbine to find out the most important factors.

**Keywords:** Power plant; Steam turbine; Gaussian Process; Cross-correlation relations; Simulation data

### 1. Introduction

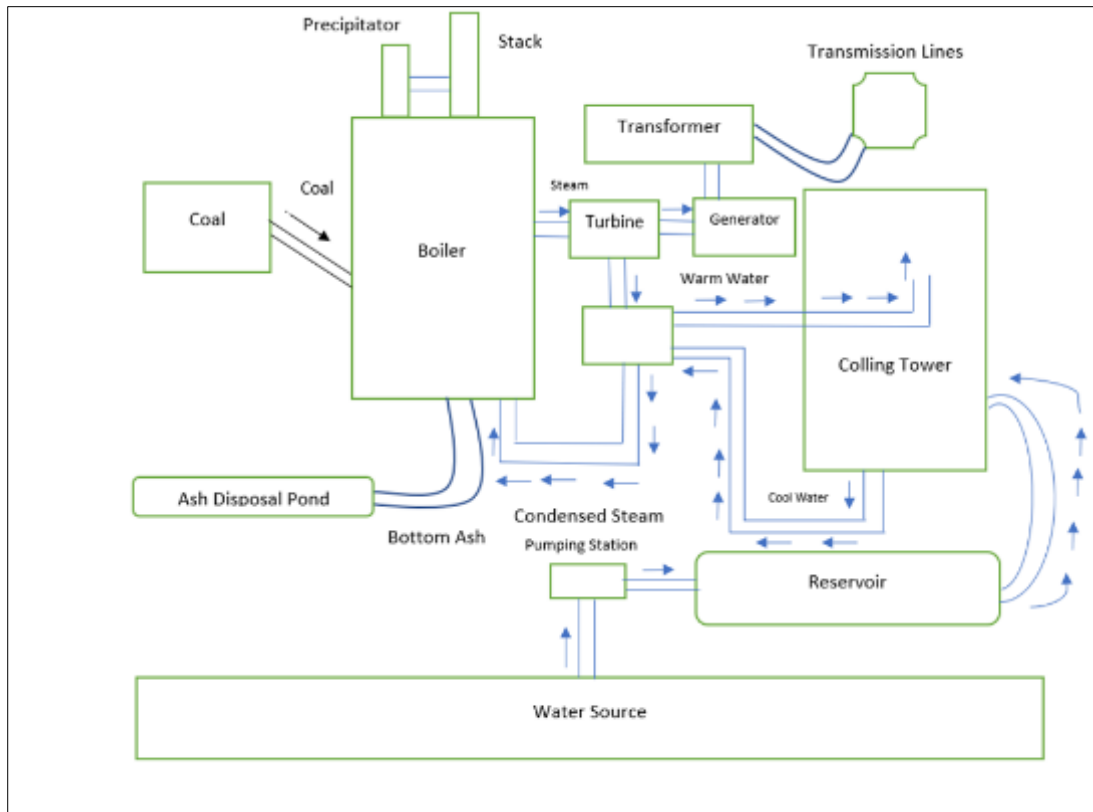
Power plants are usually classified according to their source of power, for example nuclear, coal, combined cycle, solar, steam cycle, etc. Steam cycle power plants are among the most heavily used type of power plants around the world which is based on the Ranking cycle. The cycle starts with a boiler where water is converted into steam. Steam is the main source of power for the steam turbine power plant. We have shown a schematic view of a steam turbine power plant in Figure 1. The cycle begins with a boiler that is using coal to convert the water into steam. The produced steam is then moved to the turbine and propelled to the shafts of these turbines. There are multiple turbines located in series for the effective generation of power out of the plant. Mainly three types of turbines are considered in the power plant based on the incoming steam pressure namely high, intermediate, and low-pressure turbines. These rotated turbines then generate electricity which is the final user output through the generator and transformer. During this process steams are recycled back to water due to reduced temperature and moved to the cooling tower. In this ranking cycle stage, a water source is employed to provide enough water through a pump for the plant to generate its required demand for power. The condensed water then moves back to the boiler and starts the process recursively. This is the basic coal-based steam turbine power plant working principle whereas there may be more added features such as reheater, feedwater heaters, moisture separators, etc. based on the requirement and to improve the cycle efficiency.

During the stage when steam visits the turbine sections, its temperature drops which reduce its pressure, and thus ultimately the efficiency of the low-pressure turbine reduces. A reheater is used to heat the steam to increase its temperature with a minimal amount of additional heat at the expense of extra power gained through the low-pressure turbine. The necessity of the moisture removers is also immense as it removes the droplet in the steam which can

\* Corresponding author: Mithun Ghosh

Department of Systems and Industrial Engineering, University of Arizona, Tucson, Arizona, USA.

damage the turbine blades. Feedwater heaters act as heat exchangers which can be open or closed design. The functionality of the feedwater is to heat the portion of post-condenser water steam, coming from the high-pressure turbine and send it to the boiler. This reduces the activity of the boiler by this process.



**Figure 1** Schematic view of the coal-based steam turbine

In Figure 1, a series of turbines, named after the incoming pressure of the steam, can be connected in series to increase the efficiency of the power plant model. In Figure 2, we give an overview of the turbine parts of the power plant with three turbines, namely, high pressure (HPT), intermediate pressure (IPT), and low pressure (LPT) turbines. These turbines are connected in series. As their name suggested, these turbines are different in terms of efficiency in generating power because of different pressure, temperature intakes, and also their internal structure.



**Figure 2** Turbines section of the power plant

The performance of the power plant is evaluated in different operational conditions. In order to expand the efficiency of the steam turbine power plant, different complex features and multiple expansion stages of the turbine are considered. This makes the prediction performance of the generated power more difficult as it adds uncertainty. Thus, to be more precise in power plant production, it is utmost to have a highly accurate and reliable predicting model. Also, a simple model is not enough to capture the nonlinearity and also correlations between turbines. Therefore, capturing the nonlinear dynamics of a steam turbine power plant is not a trivial task while the accuracy of nonlinearity modeling is imperative to gain a true understanding of the plant's dynamics. The developed nonlinear model has certain advantages in performing real-time simulations, control system design synthesis, and capturing the desired states [1].

There is a broad range of proposals for the dynamics of steam turbines, categorized from simple mapping algorithms from inputs to outputs to highly complex mathematical formulations [2-4]. A simple mapping algorithm is not sufficient most of the time to describe the dynamics of the power plant where many intermediate important variables are excluded [5]. Thus, due to this inaccuracy, these simple models are infeasible in terms of control dynamics, and it is imperative to have a certain degree of precision to improve the overall control performance [6].

One of the popular approaches to modeling the dynamics of the systems is to develop mathematical modeling from the observed data obtained from the real system performance. This is called the identification technique [7]. Though the ideal target without any excitation or disruption is the system identification during the normal operating condition, however, excitation or disruption is required when observed data are used for identification in most situations. This calls for soft computing methods such as parametric models to calibrate the model parameters within its operating condition range.

Genetic Algorithm (GA) is one of the heavily used optimization algorithms in the field of the mathematical-based model with observed data in the power plant dynamics. The advantage of the algorithm is the finding of the global optimal solution for the parameters of the model with complicated dynamics of system consideration [8]. Genetic algorithms are better suited for the optimization of factor vectors. However, if we try to optimize a vector of continuous variables, we will face a tremendous loss of performance.

Gaussian Process (GP) is a popular approach that is used heavily in the machine learning community. GP was primarily used in geo-statistics to capture the spatial correlations in the model. GP is a probabilistic model which gives a measurement of the uncertainty quantification also. The covariance of the model is flexible enough to capture the nonlinearity of any system. The non-stationary covariance can also capture the nonstationary of the model, especially when there are sudden or abrupt changes in the dynamics [9,18]. GP is also the best linear unbiased estimator. The cross-correlation feature in the multivariate GP (MGP) will be helpful to model the multi-output systems. MGP is just a direct extension of GP where we have multiple correlated outputs. Likewise, in our steam turbine model, we have three types of turbines and thus we have a trivariate GP model. The cross-correlation between these GPs is constructed by Hypersphere Decomposition (HD) which guarantees the validity of the covariance formulation [10,14]. MGP helps to capture more information from the data than univariate GP by lending information from the other levels. Thus, we will use this transferring knowledge framework to improve our model accuracy.

In Section 2, we first develop a Simulink model of the steam turbine power plant from the system dynamics based on the semi-empirical mathematical model described in [8]. The thermodynamics state conversion is also considered in this Simulink model. Then the simulation data is generated from these Simulink models of the turbine. For the design point generation, a design of experiment criteria is being followed. In Section 3, the simulation model is compared with the mathematical model. In Section 4, the conclusion and future direction of the work are discussed.

---

## 2. Material and methods

### 2.1. Simulink Modelling

The imperative necessity to capture the dynamics of the real-world power plant and the unavailability of real-world data motivated us to create a Simulink model where it is easier to capture the dynamics of a nonlinear system. To build a Simulink model, we need to develop the condition and the operating criteria for each element of the power plant. While this task requires a lot of labor, a Simulink model consisting of three turbines is built based on the literature reviews. The overall Simulink structure of the powerplant is given below in Figure 3. The powerplant model consists of three turbines namely: high-pressure Turbine (HPT), intermediate-pressure Turbine (IPT), and low-pressure Turbine (LPT), and based on the mathematical equation developed in [8]. Their Simulink modeling schema for these turbines is described in Figures 4, 5, and 6.

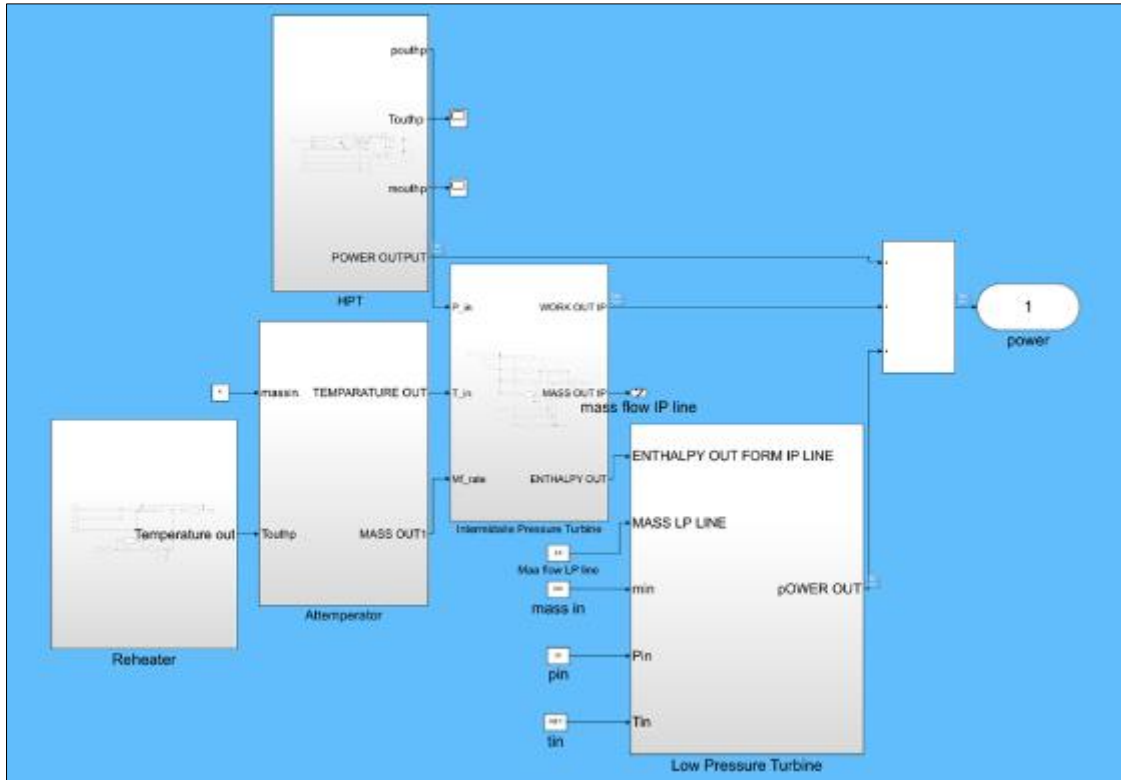


Figure 3 Simulink model of the Power Plant

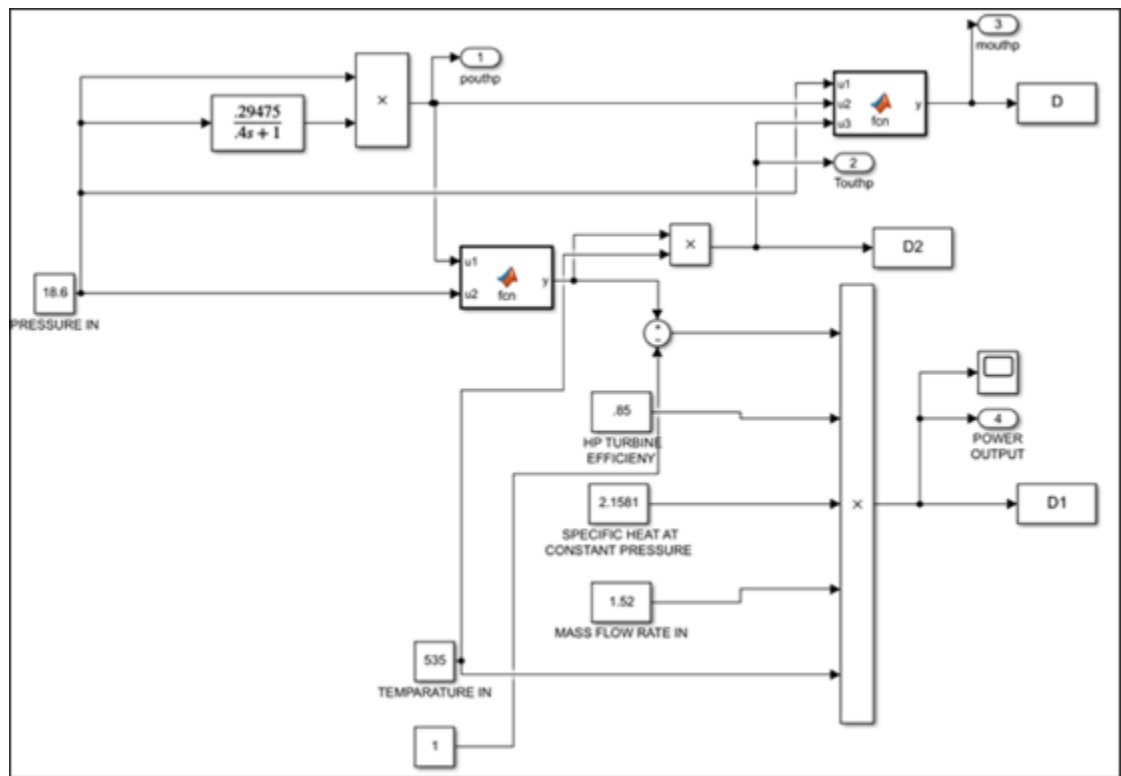
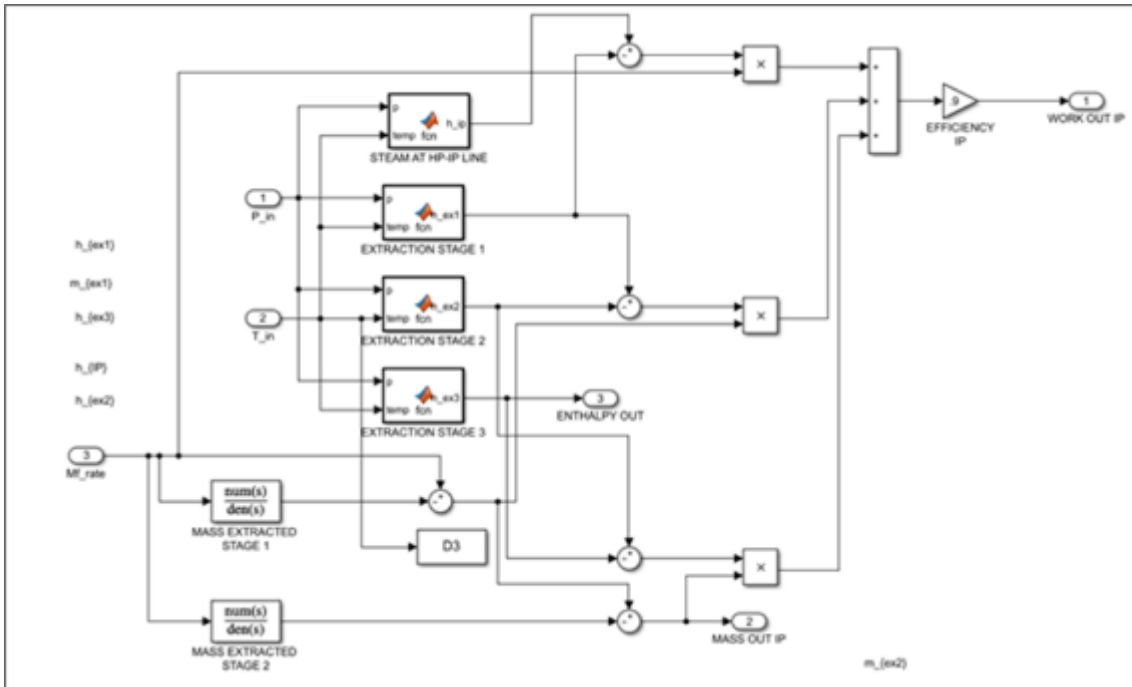
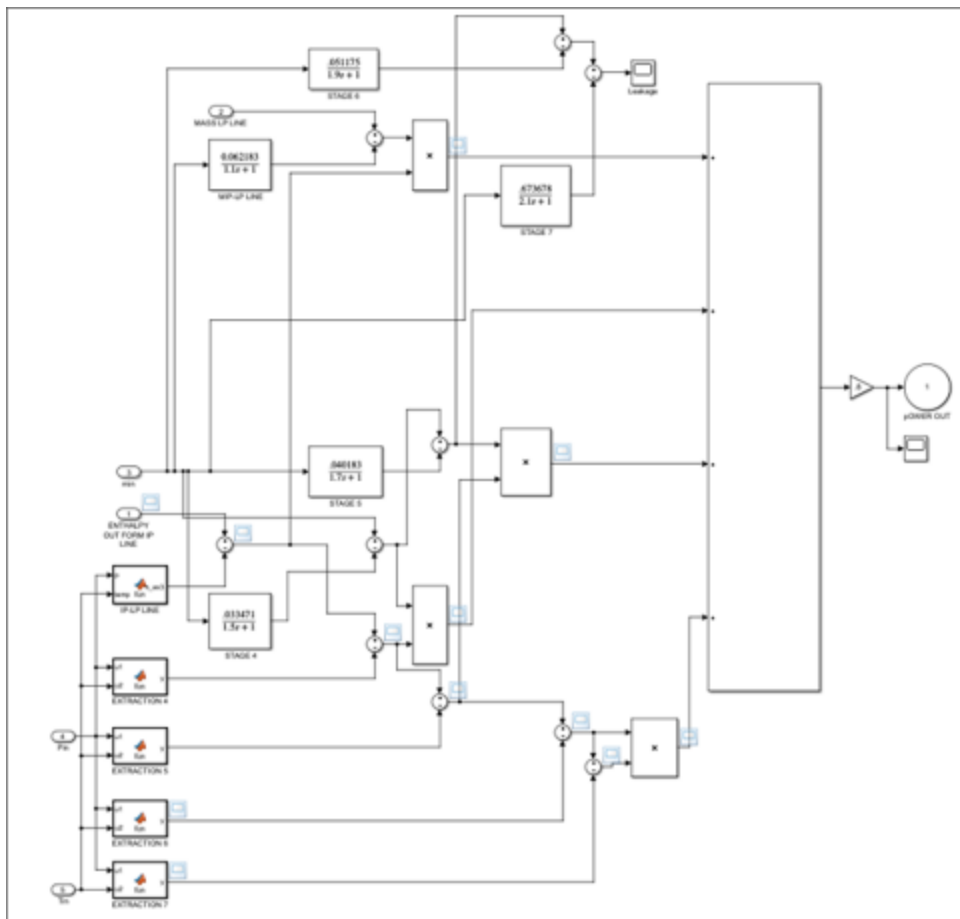


Figure 4 Simulink model of the high-pressure turbine (HPT)



**Figure 5** Simulink model of the intermediate-pressure turbine (IPT)



**Figure 6** Simulink model of the low-pressure turbine (LPT)

### 2.2. Mathematical modeling

Let's assume for a  $K$  number of turbines, we have the Simulink model for  $M$  sample runs. Then the  $i$ th ( $i=1,2,\dots,K$ ) turbine of the  $m$ th ( $m=1,2,\dots,M$ ) sample has  $N_i$  observation. The output of the  $i$ th turbine for the  $m$ th sample run of the  $j$ th combination of the variable can be modeled as the following MGP formula [11-12]:

$$\mathbf{y}^m(\mathbf{x}) = \begin{bmatrix} y_1^m(\mathbf{x}) \\ y_2^m(\mathbf{x}) \\ \vdots \\ y_K^m(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1(\mathbf{x})^T \boldsymbol{\beta}_1 \\ \mathbf{g}_2(\mathbf{x})^T \boldsymbol{\beta}_2 \\ \vdots \\ \mathbf{g}_K(\mathbf{x})^T \boldsymbol{\beta}_K \end{bmatrix} + \begin{bmatrix} \varepsilon_1(\mathbf{x}) \\ \varepsilon_2(\mathbf{x}) \\ \vdots \\ \varepsilon_K(\mathbf{x}) \end{bmatrix}$$

$$= \mathbf{G}(\mathbf{x})^T \boldsymbol{\beta} + \boldsymbol{\varepsilon}(\mathbf{x}), \tag{1}$$

Where  $\mathbf{x} \in R^l$ , the regression function for the  $i$ th turbine can be defined as

$$\mathbf{g}_i(\mathbf{x}) = [g_{i1}(\mathbf{x}), g_{i2}(\mathbf{x}), \dots, g_{iq_i}(\mathbf{x})]^T$$

And the corresponding regression coefficient is

$$\boldsymbol{\beta}_i = [\beta_{i1}, \beta_{i2}, \dots, \beta_{iq_i}]^T.$$

The regression functions can be in any order namely first-order linear function, second-order linear function, or some spline function. The term  $\mathbf{g}_i(\cdot)^T \boldsymbol{\beta}_i$  together forms the fixed part of the MGP model in Eq. (1). The  $K$ -variate error term  $\boldsymbol{\varepsilon}(\mathbf{x})$  is taken to be stationary MGP with an option to be expanded for the nonstationary MGP if needed. The mean of the MGP is taken as zero mean and with covariance function of the following

$$V_{i,n}(\mathbf{x}, \mathbf{x}') = \text{cov}(\varepsilon_i(\mathbf{x}), \varepsilon_n(\mathbf{x}')) = \sigma_i \sigma_n C_{i,n}(\mathbf{x}, \mathbf{x}'),$$

Where  $C_{i,n}(\mathbf{x}, \mathbf{x}')$  represents the overall correlation value between the  $i$ th turbine with input  $\mathbf{x}$  and  $n$ th turbine with the input  $\mathbf{x}'$ ,  $\sigma_i^2$  and  $\sigma_n^2$  are variance in  $i$ th and  $n$ th turbine's power generation, respectively. For flexibility, we can also assume the nonstationary covariance function which offers more modeling flexibility at the expense of more extra parameters [10]. With  $K=1$ , the MGP formulation in Eq. (1) degrades to the single variate GP formulation.

The standard GP model considers modeling for each turbine independently without borrowing any information from other turbines. The loss of information is enormous when the system is complex and the number of observations is scarce. The MGP formulation will help us to capture the relationship between various turbine dynamics which is ignored in the literature as of now. The flexibility of the MGP or GP covariance is an important advantage over other methods. We consider a flexible cross-correlation structure that was proposed by [13], which is given by:

$$C(\mathbf{x}_i, \mathbf{x}_n) = \sigma_i \sigma_n T_{in} \frac{\exp\{-(\mathbf{x}_i - \mathbf{x}_n)^T (\boldsymbol{\Phi}_i^{-1}/2 + \boldsymbol{\Phi}_n^{-1}/2)^{-1} (\mathbf{x}_i - \mathbf{x}_n)\}}{|\boldsymbol{\Phi}_i/2 + \boldsymbol{\Phi}_n/2| (\boldsymbol{\Phi}_i^{-1}/2 + \boldsymbol{\Phi}_n^{-1}/2)^{l/4}},$$

Where; the smoothness of the model is captured by the parameter:  $\boldsymbol{\Phi}_i = \text{diag}(\varphi_{i,1}, \varphi_{i,2}, \dots, \varphi_{i,l})$ ,

The cross-correlation between turbines  $i$ th and  $n$ th is represented by  $T_{i,n} \in [-1, 1]$ .

The overall dimension of the  $\mathbf{T}$  matrix is  $K \times K$  and  $\mathbf{T}$  is a positive definite with unit diagonal elements (PDUDE).  $T_{i,n}$  denotes the cross-correlation value between the  $i$ th and  $n$ th levels,  $T_{n,n} = 1$ . Hypersphere Decomposition (HD) is used to construct the  $\mathbf{T}$  matrix which helps it to produce a valid cross-correlation.

The construction of the  $\mathbf{T}$  matrix needs extra care due to positive definite and symmetry constraints of the covariance. To formulate a positive definite matrix with a unit diagonal element (PDUDE), references [11,14] introduced some decomposition techniques. At first,  $\mathbf{T}$  is decomposed into Cholesky decomposition such that:

$$\mathbf{T} = \mathbf{F}\mathbf{F}' \tag{2}$$

Where;  $\mathbf{F} = \{F_{rs}\}$  is a  $k \times k$  lower triangular matrix with positive diagonals. The elements of  $\mathbf{F}$  are then formulated in the spherical coordinate systems in the following way:

$$F_{rs} = \begin{cases} 1, & r = s = 1, \\ \cos(\omega_{rs}), & 1 \leq r < K, s = 1, \\ \cos(\omega_{rs}) \prod_{q=1}^{s-1} \sin(\omega_{rt}), & 1 \leq r < K, 1 < s < r. \\ \prod_{q=1}^{s-1} \sin(\omega_{rt}), & 1 \leq r < K, s = r, \end{cases}$$

Where;  $\omega_{rs} \in (0, \pi)$  which is collectively denoted by  $\Omega = \omega_{rs}\{r > s\}$ . Thus, matrix  $\mathbf{T}$  has to be constructed sequentially. Because  $\omega_{rs} \in (0, \pi)$ , the entries in  $\mathbf{T}$  are in between  $[-1,1]$  which represents correlation across different profiles. The mapping here is one-to-one between the PDUDE matrix and  $\omega$  and for any  $\omega$  we will always get PDUDE. The number of elements in matrix  $\mathbf{T}$  is  $\frac{K(K-1)}{2}$  for a  $K$  variate turbine. Thus, with  $\mathbf{T}$  we are constructing cross-correlation or between turbine correlation.

### 2.3. Factor screening and optimization procedure

We use  $L1$  regularization to the fixed term to screen out any factor that is not contributing to the modeling. We add this penalty term into the log-likelihood function where we estimate the parameter value. Factors, with less importance, will be forced to not influence by producing a sparse solution.

For the estimation of the parameter, we maximized the log-likelihood function. Considering the observation in the  $m$ th sample of the levels,  $\mathbf{y} = [\mathbf{y}_1; \dots; \mathbf{y}_k]$ , then the penalized log-likelihood function which is to be maximized:

$$\begin{aligned} & \log L(\boldsymbol{\beta}, \boldsymbol{\sigma}, \boldsymbol{\Phi}, \mathbf{T}, \sigma_{\epsilon}^2; \mathbf{y}_1, \dots, \mathbf{y}_k) - \lambda |\boldsymbol{\beta}| \\ & = -\frac{1}{2} (\log |\mathbf{C}| + (\mathbf{y} - \mathbf{G}\boldsymbol{\beta})^T \mathbf{C}^{-1} (\mathbf{y} - \mathbf{G}\boldsymbol{\beta})) - \lambda |\boldsymbol{\beta}| + \text{constant} \end{aligned} \tag{3}$$

Where;  $\mathbf{G} = \text{blkdiag}(\mathbf{g}_1, \dots, \mathbf{g}_k)$  with the dimension of  $N_i \times q_i$  of  $i$ th block being  $\mathbf{g}_i = [\mathbf{g}_{(i,1)}, \dots, \mathbf{g}_{(i,N_i)}]^T$ .  $\mathbf{C}$  is the covariance matrix.  $\boldsymbol{\Phi} = [\Phi_1; \dots; \Phi_k]$  are the roughness parameters.  $\lambda$  is a penalty term. High values of  $\lambda$  drag most of the coefficients,  $\boldsymbol{\beta}$  to zeros which implies that there is not much importance of the respective  $\mathbf{x}$ 's and vice versa for the low value of  $\lambda$ . All the parameters are derived by maximizing the log-likelihood function.

### 2.4. Prediction

The prediction for an untried input can be constructed with the estimated parameter from the proposed approach. The prediction formula for an untried point  $\mathbf{x}_0$  is:

$$\hat{\mathbf{y}}(\mathbf{x}_0) = \mathbf{g}(\mathbf{x}_0)^T \hat{\boldsymbol{\beta}} + \mathbf{c}^T(\mathbf{x}_0) \mathbf{C}^{-1} (\mathbf{y} - \mathbf{G}\hat{\boldsymbol{\beta}}) \tag{4}$$

Where;  $\mathbf{c}(\mathbf{x}_0) = \mathbf{C}(\mathbf{x}_0, \mathbf{x})$ .

### 2.5. Design of Experiment

A space-filling design criterion is used to collect the observation from the Simulink model. One of the most popular methods, latin hypercube sampling (LHS) is used to generate a trivariate training dataset for the power plant model for 50 observations for each turbine with 30 sample runs. LHS has the advantage of being a space-filling design with multidimensional input. It produces a near-random observation in such a manner that we can explore most of the input space. We use six-dimensional LHS with minimax criteria to ensure the exploration of most of the area in space for the input designs. Similarly, another 50 observations and 30 replications are generated for the test dataset. From the literature, the selected six input parameters that contribute most to the power plant and their selected range for the simulation are described below:

**Table 1** Inputs with range

Variable	Input	Range
1	Pressure	35-10 MPa
2	Temperature	2000-500 K
3	Mass flow	3-2.2 kg/s
4	Electric grid frequency	60-50 Hz
5	Number of blades	20-5
6	Boiler temperature	650-550 K

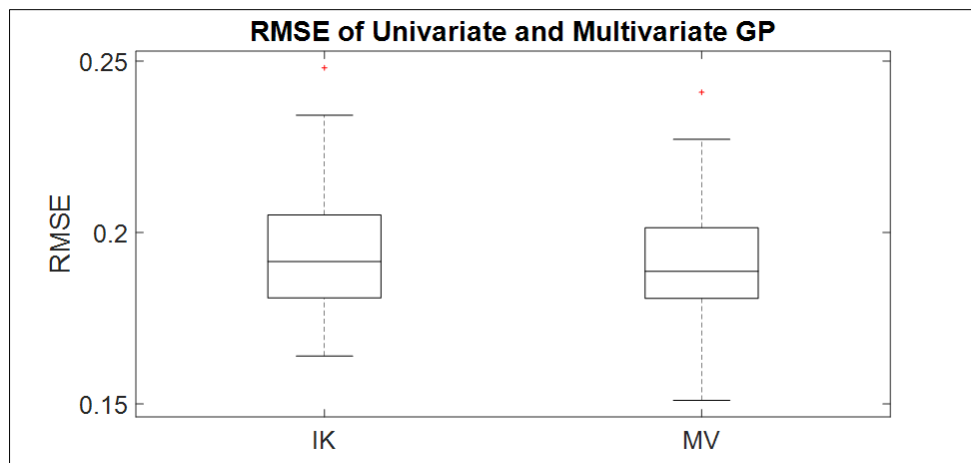
As this is a dynamic model, we will get different outputs in the simulation at different time steps. But at a certain time, output reaches steady states when its values do not change with time. We record that value for each simulation and treat it as the final output value in the modeling. For brevity, we also name the input with the variable number which is the left-most column in Table 1.

### 3. Results and Discussion

We fit our model with the univariate or independent GP and the multivariate GP framework. The RMSE value for each model is described in Figure 7. Both of the models are in close range with low RMSE which proves the predictive capability of GP. The MGP model outperformed the GP model slightly by capturing the cross-correlation correctly between the turbines. This gives the MGP model edge over the Univariate GP (UGP) model. If we can analyze the cross-correlation parameters in Table 2. It seems the HPT and IPT are correlated strongly with the value of 0.752. Whereas, other correlations between the turbines are low positive. It is as expected. With the increase in the steam temperature and pressure values, these correlation parameters should increase. If there were no pressure and temperature loss, these turbines would have ideal cross-correlation of 1.

**Table 2** The cross-correlation structure of the turbines

1	0.7592	0.0575
0.7592	1	0.0453
0.0575	0.0453	1

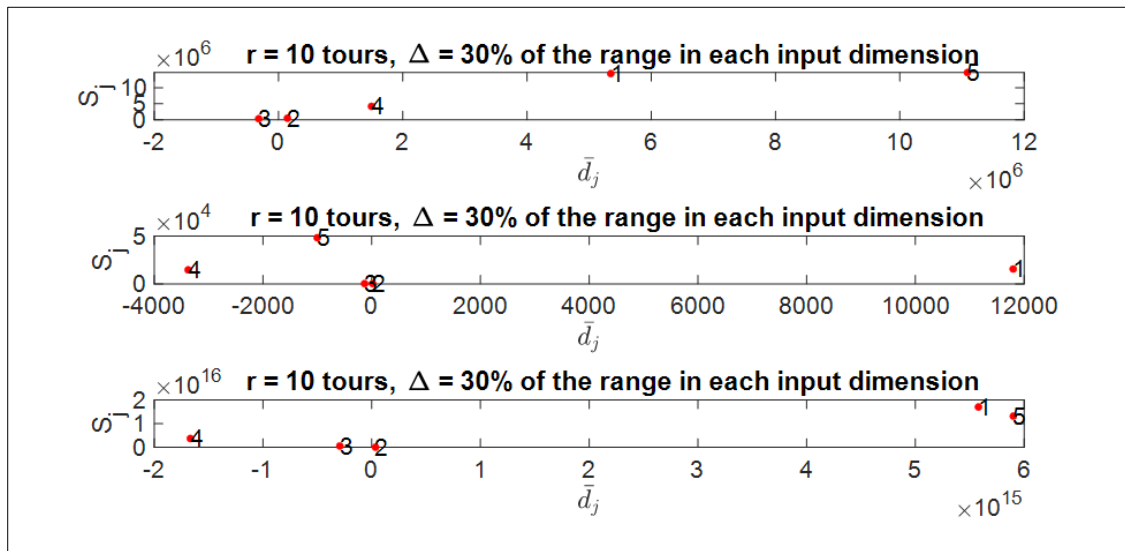


**Figure 7** Two random realizations of the turbine model with the actual and predicted value



We also perform the sensitivity analysis of the MGP model by the Elementary Effect (EE) method. EE method quantifies the noninfluential inputs in the computationally costly models. It ranks the inputs in order of importance and helps to discard the less important ones when we have a large number of inputs. The details can be found in [15] and [16].

For 10 tours and 30% in variables of its range are used for the analysis. In Figure 8 the sensitivity of the three turbines is described where we can see the plot of mean vs variance of the parameters. Low values of both mean and variance correspond to a non-influent input. The corresponding input name for the variable number is shown in Table 1. Variables 6 have a too large mean and variance which is why it is excluded in the plot. Most of the variables in these turbine models have high mean and variance which redefine the high nonlinearity of the turbine model in its input and output relations. Especially variables 1,5 and 6 which are pressure, number of blades, and boiler temperature are more influential than others.



**Figure 8** Sensitivity analysis of the turbine model by EE algorithm. From the above, the plots are for HPT, IPT, and LPT correspondingly

#### 4. Conclusion

The power plant model is highly nonlinear. Predicting the model output power is an extremely important and difficult task without a complex machine learning algorithm. MGP performs very well to capture the original pattern of the output power. To capture the intrinsic behaviors of the data, MGP plays a great role as it allows different types of turbines to capture information from other types of turbines in the cross-correlation structure. Sensitivity analysis also showed us the input-output nonlinear interactions. In the future, a unified MGP model for the whole steam turbine power plant can be built such that we can have a total output power in the output end. Also, we can think of a dependent GP structure such that we can build a model for the time series data not just the steady-state values of the output. Also, this model can be employed and tested if we can get the real data in the future. An ensemble framework of neural net [17] and GP will also improve the model performance to deal with the nonstationarity of the complex model.

#### Compliance with ethical standards

##### *Acknowledgments*

The authors are grateful to the anonymous referees for helpful suggestions that improved the presentation of the article.

##### *Disclosure of conflict of interest*

There is no conflict of interest.

---

**References**

- [1] IEEE PES Working Group. Hydraulic Turbine and Turbine Control Models for System Dynamic. IEEE Transaction on Power System. 1992, 7; 167-174.
- [2] Anglart H, Andersson S, Jadrny R. BWR steam line and turbine model with multiple piping capability. Nuclear Engineering, and Design. 1992, 137; 1-10
- [3] Hannett LN, Feltes JW. Testing and model validation for combined-cycle power plants. IEEE Power Engineering Society Winter Meeting. 2001, 2; 664-670.
- [4] Ray A. Dynamic modeling of power plant turbines for controller design. Application of Mathematical Modelling. 1980, 4; 109-112.
- [5] Habbi H, Zelmata M, Ould BB. A dynamic fuzzy model for a drum boiler-turbine system. Automatica. 2003, 39; 1213-1219.
- [6] Forssell, Ljung L. Closed-loop identification revisited. Automatica. 1999, 35; 215-241.
- [7] Fleming PJ, Purshouse RC. Evolutionary algorithms in control systems engineering: a survey. Control Engineering Practice, 2002, 10; 1223–1241.
- [8] Ghosh M, Li Y., Zeng L et al. Modeling multivariate profiles using Gaussian process-controlled B-splines. IISE Transactions. 2020, 1-12.
- [9] Li Y, Zhou Q., Huang, X., & Zeng, L. Pairwise Estimation of Multivariate Gaussian Process Models With Replicated Observations: Application to Multivariate Profile Monitoring. Technometrics, 2018, 60(1), 70-78.
- [10] Li, Y, & Zhou Q. Pairwise meta-modeling of multivariate output computer models using nonseparable covariance function. Technometrics. 2016, 483-494; 58(4).
- [11] Chaibakhsh, A., & Ghaffari, A. A nonlinear steam turbine model for simulation and state monitoring. In Power and Energy System Conference. 2008, 347-352.
- [12] Fricker TE, Oakley JE, Urban NM. Multivariate gaussian process emulators with nonseparable covariance structures. Technometrics. 2013, 55; 47–56.
- [13] Qiang Z, Peter ZG, Shiyu Z. A simple approach to emulation for computer models with qualitative and quantitative factors. Technometrics. 2011, 266-273; 53(3).
- [14] Rebonato R, Jackel P. The most general methodology to create a valid correlation matrix for risk management and option pricing purposes. The Journal of Risk. 1996, 2, 17–28.
- [15] Morris MD. Factorial sampling plans for preliminary computational experiments. Technometrics. 1991, 33; 161–174.
- [16] Campolongo F, Cariboni J, Saltelli A. An effective screening design for sensitivity analysis of large models. Environmental Modelling and Software. 2007, 22; 1509–1518.
- [17] Ghosh, M., Hassan, S., & Debnath, P. Ensemble Based Neural Network for the Classification of MURA Dataset. Journal of Nature, 2021, 4, 1-5.
- [18] Ghosh, M., Li, Y., Zeng, L., Zhang, Z., & Zhou, Q. Modeling multivariate profiles using Gaussian process-controlled B-splines. IISE Transactions, 2021, 53(7), 787-798.