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Implementation of image compression based on singular value decomposition

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Abstract

Analyzing big data amount and the limitation of the storage data devices and the rate of data is one of the most critical issues in data processing and transferring. In this research, one of the essential approaches for image compression is proposed. Singular value decomposition (SVD) is a highly effective mathematic technique which is used for the reduction process applied to redundant data in order to minimize the required storage space or transferring channel. The main idea of this work is divided into two main phases. The first phase is explained the (SVD) computational steps approach in detailed while the second phase is described the result of the applying (SVD) in the field of image compression. The achievement results of this experiment show a powerful controlled technique depend on the desired rank of a decomposed image in order to achieve a compression result by the using of a factorized matrix on image compression. The performance of the compressed image is examined in terms of peak to signal ratio and compression ratio.

Keywords: Singular Value Decomposition; Image Compression; Peak signal to noise ratio; Matrix Factorization

1. Introduction

Nowadays, images commonly used for different computer applications. The size factor of digital images is considered as an important issue in terms of storing and transferring [1],[2]. One possible technique to overcome this problem is to use data compression technology which is to deal with an image as a matrix. Considering the image as a matrix allowed performing all the mathematical operations on the image matrix. Image compression is achieved by applying singular value decomposition (SVD) techniques on the image matrix. The advantages of using SVD are the characteristics of energy compacting and compression it adapts to the ability of the image to change locally statistically. Besides, SVD can be executed on an arbitrary square of m×n size, reversible and irreversible.

(SVD) is a powerful technique used to minimize data for storage and transmission. (SVD) is considered to be an essential topic for many famous mathematicians in linear algebra. In addition to image compression SVD involved in much practical application. Because of the ability of SVD to executed on any real matrix. Its factorized matrix A into three orthogonal matrices matrix (U), matrix (S), and matrix (V) in order to obtain the final matrix $A = USV^T$. Many applications used SVD in image processing such as a watermarking technique for security requirement [3], soft sensor for smart system [4], modulated in de-noising algorithm partial discharge (PD) signals [5], and measure turbulent transport coefficients in a simulation of the turbulent interstellar in large scale dynamo [6]. In this research, the ability of SVD for compression image will be described and tested. The structure of this work will be as follow: the brief related work is explained. Then the theory of the SVD is explained and applied in mathematical form. Finally, the implantation of the compression image is carried out further the quality factors of the achieved results is determined and discussed.

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2. Related Work

The compression process in the image can be defined as it is the process to remove the redundant data. The correlation between neighbor pixels causes the redundant data in the image. The goal of data to be compressed is to illuminate the required numbered of bits which are needed to form the data. In order to simplify the process need to be applied to image such as storing, enhancement, segmentation, and transferring the compression processes become very important [7].

The classification of image compression can be divided into the main approaches. The first one is the free loss compression method which is guarantee no loss will be achieved during the compression process. In contrast, the second method is the lossy compression techniques which are mean that the images will be losing some of its information during the compression process. It can be explained that the unnecessary data in the image depend on the type of the image application. On other terms, the data in a medical image can be considered very valuable in order to make medical staff take a right and specific decision. In contrast, some data in personal image in the background can be ignored.

The quality measurement of the compression techniques is measured by determining the peak signal to noise ratio. The maximum signal to noise ratio is used to compute the amount of noise obtained by lossy compression of the image. However, the visualization of the observer is must be considered [8].

While the (SVD) can be used for coding image [9][10], there are many techniques for image coding methods. For classification of coding techniques, there are different classification methods from different perspectives. According to whether the original image can be completely restored, the image compression coding can be divided into lossless compression coding and lossy compression coding [11]. Lossless compression requires no loss of information after information compression. That is, the original image can be reconstructed without distortion after decoding the compression coding of the image. The commonly used lossless compression coding includes Huffman coding [12] and arithmetic coding [13]. The second classification of image coding is the lossy approach, such as zero tree wavelet [14].

The development of image compression coding technology is closely linked to the needs of the required applications. The breakthrough in this field will have a profound impact on the development of communications and multimedia businesses. The next section theory of SVD will be explained.

3. Theory of Singular Value Decomposition (SVD)

SVD is a mathematical approach which depends on the linear algebra principles. The powerful of SVD comes from the ability to factorize the matrix into three sub-matrices each of these matrices has specific properties. Let the matrix (A) with dimension 2 × 2 need to be factorized. SVD is the factorization of A into three matrices such as matrix U and the transpose of the matrix (V), where U and V considered orthonormal matrices. The third part of factorization is a diagonal matrix (∑) which is inclusive of the singular values of the matrix (A). The diagonal elements of the matrix consist of singular values $\sigma_1 \geq \cdots \geq \sigma_n \geq 0$ appear in σ descending order. The numbers $\sigma_1^2 \geq \cdots \geq \sigma_n^2$ are the determined eigenvalues of ($A * A^T$) and ($A^T * A$) as explained in equation (1) [15] [16][17]:

$$A = U \sum V^T \qquad (1)$$

Where

$$U = [u_1 \ldots u_n], \Sigma = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_n \end{pmatrix}, V^T = \begin{bmatrix} V_1^T \\ \vdots \\ V_n^T \end{bmatrix} (2)$$

In order to simplify the steps for factorization matrix, the following example is applied:

Matrix (A) has a rank equal to 2, as shown in the following equation:

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}, A^T = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \qquad (3)$$

The first step is to find the results of the $A A^T = Q$ and $A^T A = W$. The following results are determined as follow:

$$Q = \begin{bmatrix} 8 & 0 \\ 2 & 0 \end{bmatrix}, W = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix} \quad (4)$$

To complete the factorization of the matrix A the eigenvalues must be calculated by determine the square roots of the computed eigenvalues, λ_1 and λ_2. The final form $\sum$ can be achieved when σ_1 and σ_2 are computed. The following steps are described:

$$|Q- \lambda I|=0 \quad (5)$$

$$|W-\lambda I|=0 \quad (6)$$

By solving equation (1) and (2) λ_1=8 , λ_2=2 are obtained. The ($\sum$) matrix now is formed as below:

$$\sum = \begin{bmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \quad (7)$$

To form the entire factorization matrix the (U) and (V) matrices is determined by solving equation (4) and (5):

$$(Q - \lambda I)\ x = 0 \quad (8)$$

$$(W - \lambda I)\ x1 = 0 \quad (9)$$

The calculated results for (U) and (V) matrices explained as follow:

$$U = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{bmatrix} \quad (10)$$

The final form of matrix (A) factorization as equation (1) described will be as the following equation:

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & 0.707 \end{bmatrix} \quad (11)$$

4. Images and SVD Compression Method

Since the grey image can be considered a matrix with dimension (n*m) pixels and a colour image consist of three matrices. SVD can be applied for an image in order to factorize it into three orthogonal matrices. This process allows us to reform the image matrix by finding the summation of more simplified matrices has rank one. The approximate of the matrix (A) can be achieved by using fewer entries amount than include in the original image. This can be achieved by control the rank of a matrix and illuminate the redundant data when $r < m$, or $r < n$ where (r) is represented the rank value of the matrix. Since the singular values always must be higher than zero value. So, by adding on the dependent terms where the value of singular is zero and will not affect the visualization image. The values at the end of the equation zero is out leaving:

$$A = \sum_i^n \sigma_i\ u_i v_i^T \quad (12)$$

Now the compression process can be achieved by illuminating more singular elements of the input matrix (A). Because of the decreasing order arrangement of singular elements, the last elements can be ignored due to the minimum effect on the image and consider it as redundant. By doing this process will lead to reducing the number of bits required to represent the image. On other terms matrix A can be written by adding only the first few terms of the series. Rank (K) matrix will be obtained by adding all the factorized matrices with rank one as equation (13) described:

$$A_{approx} = \sum_i^k \sigma_i\ u_i v_i^T \quad (13)$$

The compression goal can be achieved by controlling the limitation of (k) value which has a direct effect on the quality of the image and the compression value.

5. Results and discussion

The factorization algorithm SVD algorithm is implemented in this section using Matlab platform. The comparison of image compression will be examined in terms of image quality. The peak to noise ratio will be evaluated in order to compare the image quality for various image compression ratio [18]. Further, the rank of the SVD matrix will be changed in order to realize the effect on the compression ratio and image visualization, as shown in Figure 1.

Figure 1 Image before compression

In order to notice the visualization effect on the image after different compression rank will be selected, as explained in Figure 2.

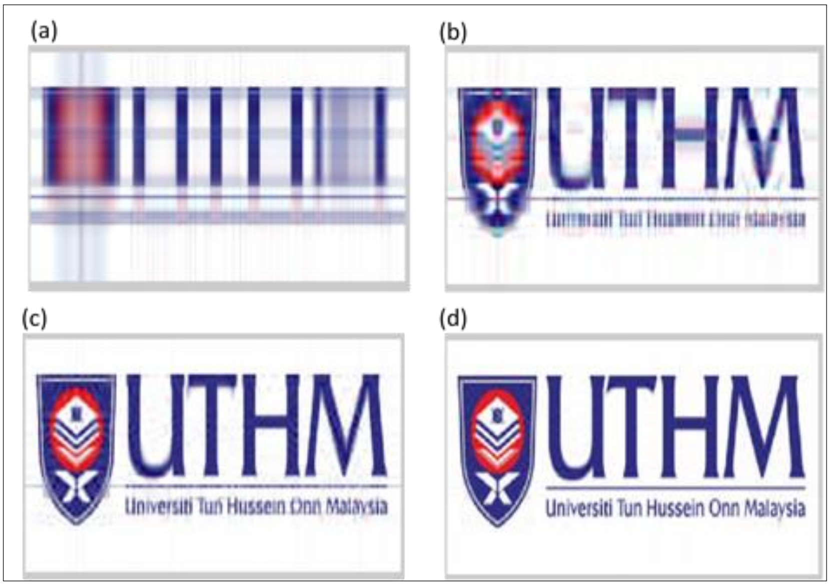

Figure 2 Compressed image for various rank: a. Rank 2 b. Rank 8 c. Rank 16 d. Rank 32

It can be noticed that the detail of compressed image loosed when the rank compressed is decreased, which is lead to high loss; however, the compression ratio will be increased as the below table is explained.

Table 1 Determined compression ratio and PSNR for various matrix rank

Matrix rank	PSNR	Compression ratio
2	18	95.9
4	19.9	47.95
8	22.8	23.97
16	25.7	11.98
32	28.9	5.9
64	33.8	2.99
128	43.1	1.99

Therefore, the value of (k) as described in equation (13) consider the main factor of the primary goal of SVD to compress the image and control the loss value from choosing the dimensions of the factorized output matrix as described in Figure 3 and Figure 4.

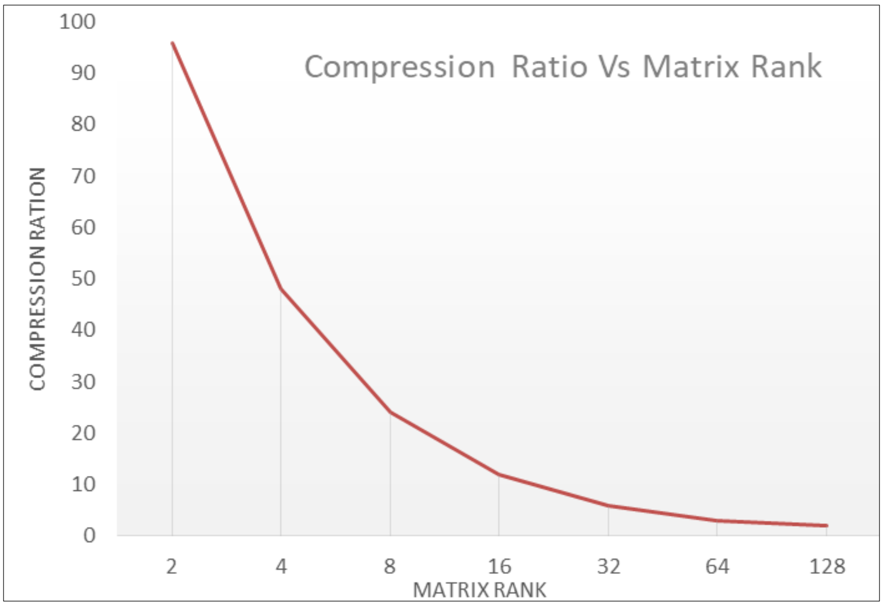

Figure 3 Evaluated compression ration versus factorized matrix rank

It can be indicated from Figure 3 that the increase in the rank of the factorized matrix that the compression ratio will be decreased at the same time the image quality will be increased. However, the size of the compressed image increased as a result of the compression ratio decrease. The second factor of the compression performance in terms of PSNR is explained in Figure 4.

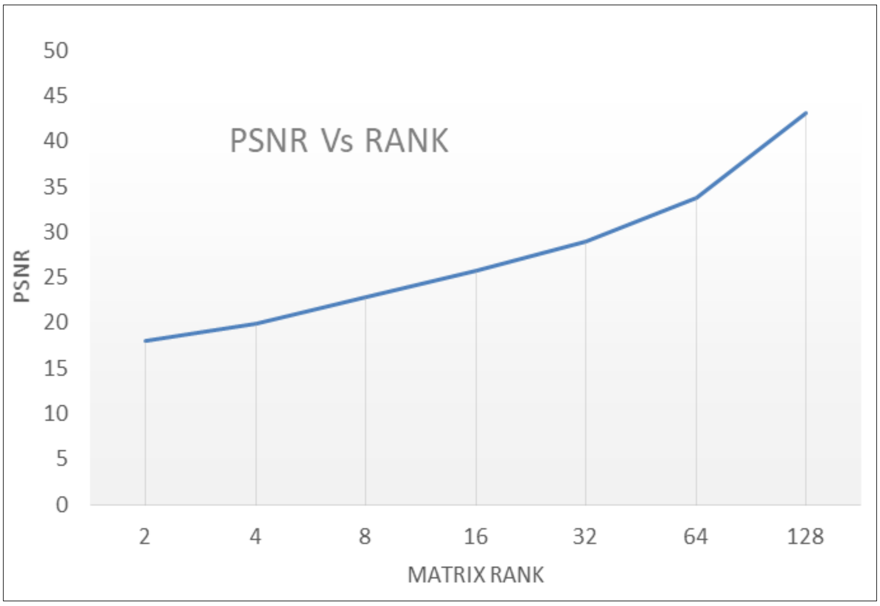

Figure 4 Peak to signal ratio compared to matrix rank

The quality of the compressed image is examined in terms of the peak to signal ration. The referenced image is the original image before compression processing. Figure 4 indicates that the rank of the matrix result from the factorization of SVD is to increase the PSNR at a high level. In other words, the image quality is increased as matrix rank will be increased.

6. Conclusion

The image compression method is proposed in this research. SVD applied in image compression was tested. The detailed steps for SVD theory are discussed and explained. The effect of the rank on the image compression and compression ratio is determined. The determined results indicated that when the output compressed image has rank equal to 2 the achieved compression ratio is 95.9 while the PSNR is 18. Therefore, the rank of the factorized image is the controller of the image compression ratio. The work can be extended to embed a neural network in order to enhance the achieved results and decrease the image losing quality.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this study.

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