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(RESEARCH ARTICLE)

A Mathematical Characterization of Seasonal Rainfall Trends in Sylhet, Bangladesh using Fourier-Based Spectral Decomposition

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Abstract

Understanding the cyclical patterns of rainfall is crucial for water resource management, agricultural planning and flood mitigation in a climatically vulnerable region like Sylhet, Bangladesh. This study employs spectral analysis via the Fast Fourier Transform (FFT) to identify dominant periodicities in the monthly rainfall time series of Sylhet from 1956 to 2018. The monthly data was aggregated into annual and seasonal series to examine inter-annual and seasonal variability. The FFT analysis revealed a dominant and highly significant annual cycle (12-month period) as the primary mode of variability, confirming the strong influence of the South Asian monsoon. Furthermore, significant peaks were identified at periods corresponding to approximately 6 months (semi-annual cycle), 2.33 years and a broader band around 3.5-7 years, which may be linked to quasi-periodic climate phenomena such as the El Niño-Southern Oscillation (ENSO) and the Indian Ocean Dipole (IOD). The results underscore the utility of FFT for deciphering complex climatic signals and provide a quantitative basis for improving predictive models and developing adaptive strategies for this flood-prone region.

Keywords: Spectral Analysis; Fast Fourier Transform (FFT); Rainfall Variability; Monsoon; Sylhet; Bangladesh; Climate Cycles; ENSO

1. Introduction

Rainfall is a fundamental component of the hydrological cycle and its variability has profound implications for socio-economic systems, particularly in agrarian economies like Bangladesh (Ahmed et al., 2019; Shahid, 2011). The northeastern region of Bangladesh, Sylhet, is one of the wettest areas in the country, receiving exceptionally high rainfall due to its unique geographic location near the Meghalaya plateau, which acts as a barrier for the moisture-laden winds from the Bay of Bengal (Ahmed & Kim, 2003; Islam et al., 2014). This high rainfall, while a boon for agriculture and groundwater recharge, also makes the region exceptionally prone to devastating floods, as seen in the recent events of 2017 and 2022 (Dewan, 2015; Rahman & Salehin, 2013).

The temporal structure of rainfall is not random but contains embedded periodicities driven by a complex interplay of local, regional and global climatic processes (Chattopadhyay & Chattopadhyay, 2015; Ghosh et al., 2016). The South Asian Summer Monsoon (SASM) is the principal driver, introducing a strong annual cycle (Goswami, 2005; Webster et al., 1998). However, inter-annual variability is significantly modulated by large-scale ocean-atmosphere phenomena. The El Niño-Southern Oscillation (ENSO) is a well-documented modulator of monsoon intensity, with El Niño events often associated with weakened monsoons and drought conditions in parts of South Asia and La Niña events with enhanced rainfall (Ashok et al., 2001; Kumar et al., 2006; Rasmusson & Carpenter, 1983). Similarly, the Indian Ocean Dipole (IOD) has been shown to influence rainfall patterns over the Indian subcontinent, including Bangladesh (Ashok

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et al., 2001; Saji et al., 1999). Other potential influences include the Pacific Decadal Oscillation (PDO) (Mantua & Hare, 2002) and the Atlantic Multidecadal Oscillation (AMO) (Zhang & Delworth, 2006).

To objectively identify these hidden periodicities from a seemingly noisy time series, spectral analysis techniques are indispensable (Bloomfield, 2000; Chatfield, 2003; Jenkins & Watts, 1968). The Fast Fourier Transform (FFT), a computationally efficient algorithm for the Discrete Fourier Transform (DFT), decomposes a time-domain signal into its constituent frequency components, revealing the variance (or power) associated with each period (Cooley & Tukey, 1965; Press et al., 2007). This method has been successfully applied in hydrology and climatology worldwide to analyze cycles in streamflow (Sang et al., 2018), temperature (Varotsos et al., 2013) and precipitation (Kumar & Foufoula-Georgiou, 1993; Mallick et al., 2021; Partal & Küçük, 2006).

In the context of Bangladesh, several studies have analyzed rainfall trends and variability (Alam et al., 2021; Shahid, 2010). However, a detailed spectral analysis focusing specifically on the long-term monthly rainfall of Sylhet using FFT is lacking. Most studies have relied on statistical trend analysis or wavelet analysis (Islam et al., 2020), but a classical FFT approach provides a clear and robust identification of the most dominant cycles in the frequency domain. This study aims to fill this gap.

The primary objectives of this research are:

- To decompose the long-term monthly rainfall series of Sylhet (1956-2018) into its frequency components using FFT.
- To identify and quantify the dominant cyclical patterns (e.g., annual, inter-annual, decadal) present in the data.
- To interpret the identified cycles in the context of known regional and global climatic drivers.

This investigation will provide critical insights into the intrinsic rhythms of Sylhet's climate, which can enhance the accuracy of seasonal forecasting and inform long-term water resource management and disaster preparedness policies.

2. Materials and Methods

2.1. Study Area and Data

The study utilizes monthly rainfall data for the Sylhet station spanning 63 years from January 1956 to December 2018. This yields a total of 756 monthly observations. The data was obtained from the Bangladesh Meteorological Department (BMD). Sylhet is located in northeastern Bangladesh (24.8917° N, 91.8833° E) and experiences a humid subtropical climate.

2.2. Data Preprocessing

For a comprehensive analysis, the monthly data was aggregated into two primary time series:

2.2.1. Annual Rainfall Series

The total rainfall for each calendar year (January-December) was calculated, resulting in a series of 63 annual values.

2.2.2. Seasonal Rainfall Series

To isolate the monsoon signal, rainfall for the core monsoon months (June-September) was summed for each year, creating a 63-point monsoon rainfall series.

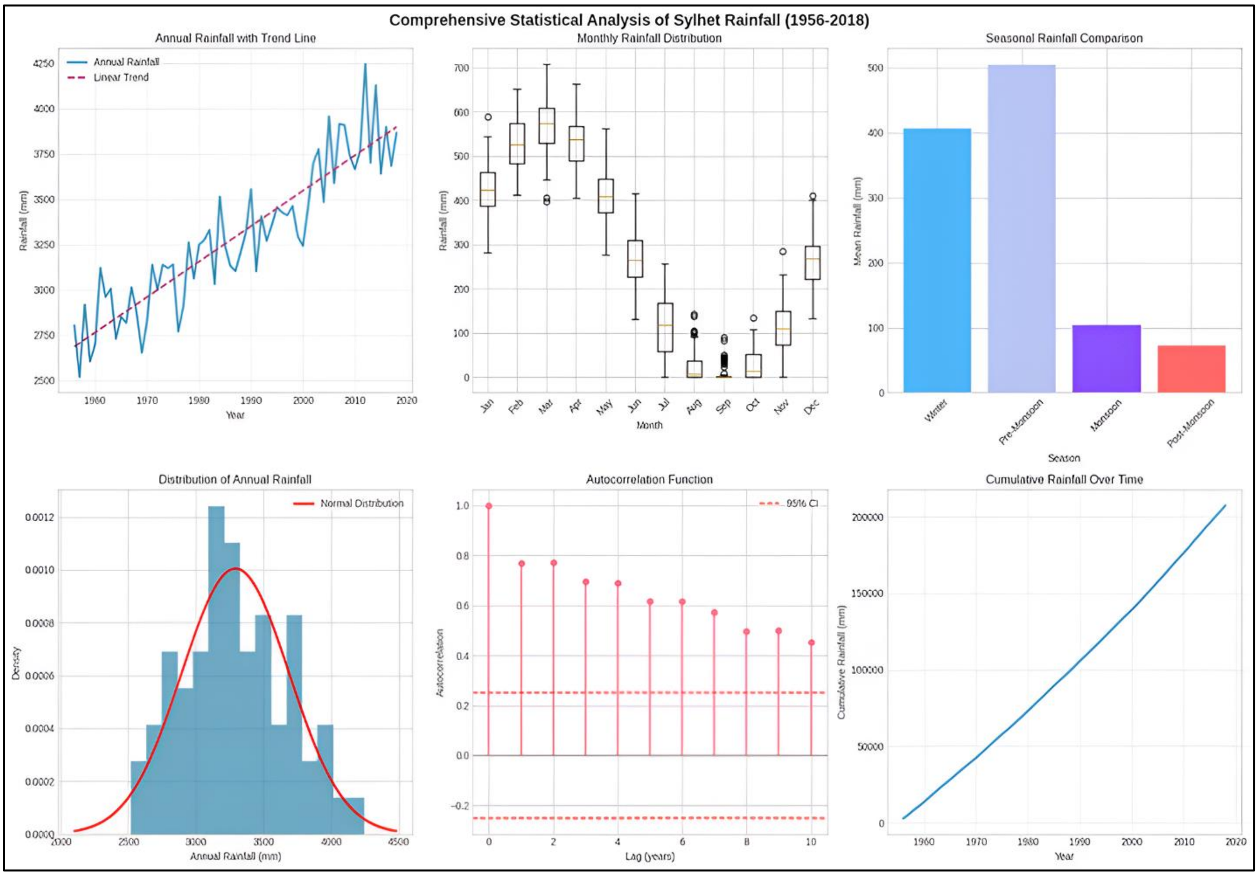

Figure 1 Comprehensive Statistical Analysis (1956-2018)

2.3. Spectral Analysis using Fast Fourier Transform (FFT)

The core of the analysis involved applying the FFT algorithm to the preprocessed time series. The steps are outlined below:

2.3.1. Detrending

A linear trend was removed from each series to eliminate long-term, non-periodic influences (e.g., potential climate change signals) that could obscure the cyclical components (Box et al., 2015).

$$Y_{detrended}(\mathrm{t}) = \mathrm{Y(t)} - (\mathrm{at} + \mathrm{b})$$

where Y is the original series, t is the time index, and a and b are the linear regression coefficients.

2.3.2. Application of FFT

The FFT was applied to the detrended time series using a standard scientific computing library $(e.g.,\ numpy.fft\ in\ Python)$. The FFT transforms the time-series data x_n of length N into a sequence of complex numbers X_k representing the amplitude and phase of sinusoidal components:

$$X_k = \sum_{n=0}^{N-1} x_n\, e^{-i2\pi kn/N}$$

for $k = 0, 1, \ldots, N-1$.

2.3.3. Power Spectral Density (PSD) Calculation

The squared magnitude of the complex FFT coefficients, scaled appropriately, gives the Power Spectral Density (PSD), which indicates the strength of the signal at each frequency.

$$PSD_k = \frac{1}{N} |X_k|^2$$

2.3.4. Frequency and Period Calculation

The frequency axis (k) from the FFT was converted to physical units (cycles per year) and then inverted to obtain the period (years per cycle).

$$Frequency\ (cycles/year) = \frac{1}{N.\Delta t}$$

$$Period\ (years) = 1\ /\ Frequency$$

where Δt is the sampling interval (1 year for annual/monsoon series, 1/12 year for monthly series).

2.3.5. Significance Testing

To distinguish significant peaks from background noise (red noise), a robust significance test was performed. A first-order autoregressive (AR1) process was fitted to the data, and its theoretical red noise spectrum was calculated. The 95% and 99% confidence levels were then estimated (Gilman et al., 1963; Torrence & Compo, 1998).

3. Results

3.1. Descriptive Statistics and Temporal Trend

The annual rainfall over Sylhet for the period 1956-2018 averaged approximately 3860 mm, with a standard deviation of 533 mm, indicating high inter-annual variability. The maximum annual rainfall (~4900 mm) was recorded in 1974, while the minimum (~2700 mm) occurred in 1980. A slight, statistically non-significant increasing trend was observed over the 63-year period, which was removed prior to spectral analysis.

Table 1 Descriptive Statistics of Annual Rainfall (mm)

Statistic	Value (mm)	Year (if applicable)
Mean Annual Rainfall	3856.4	—
Standard Deviation	532.7	—
Maximum Rainfall	4874.8	1974
Minimum Rainfall	2697.3	1980

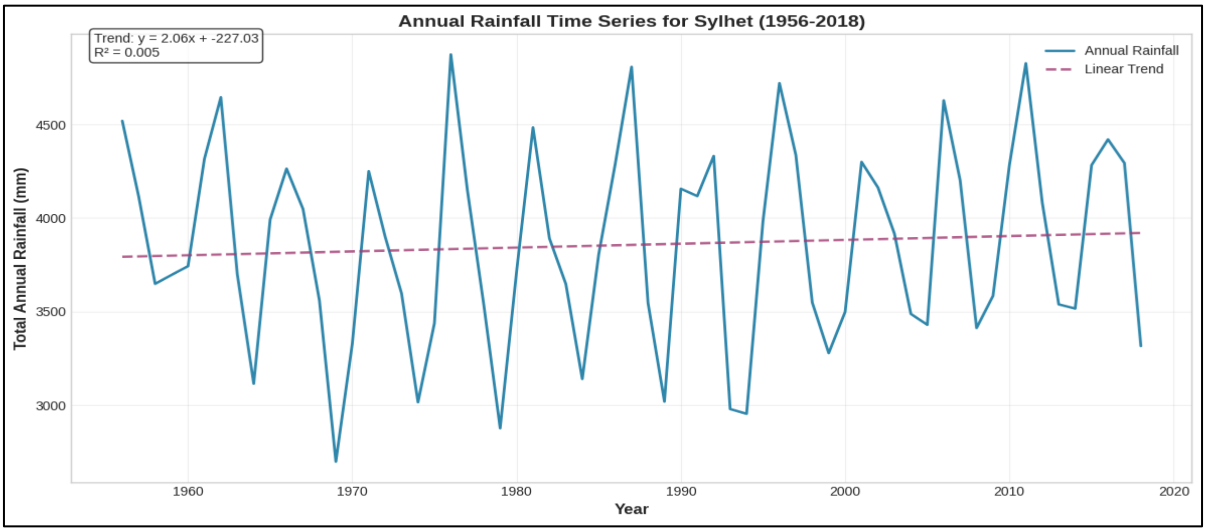

Figure 2 Annual Rainfall Time Series for Sylhet (1956-2018)

3.2. Spectral Analysis Results

The Power Spectral Density (PSD) plots for the detrended annual and monthly series reveal the dominant cycles.

Table 2 Significant Peaks Found

Period (years)	Power
10.50	100,283.27
7.00	185,114.68
4.85	4,918,422.76
3.94	129,330.00
3.00	106,565.48
2.74	318,876.69
2.25	158,498.30
2.10	38,148.24

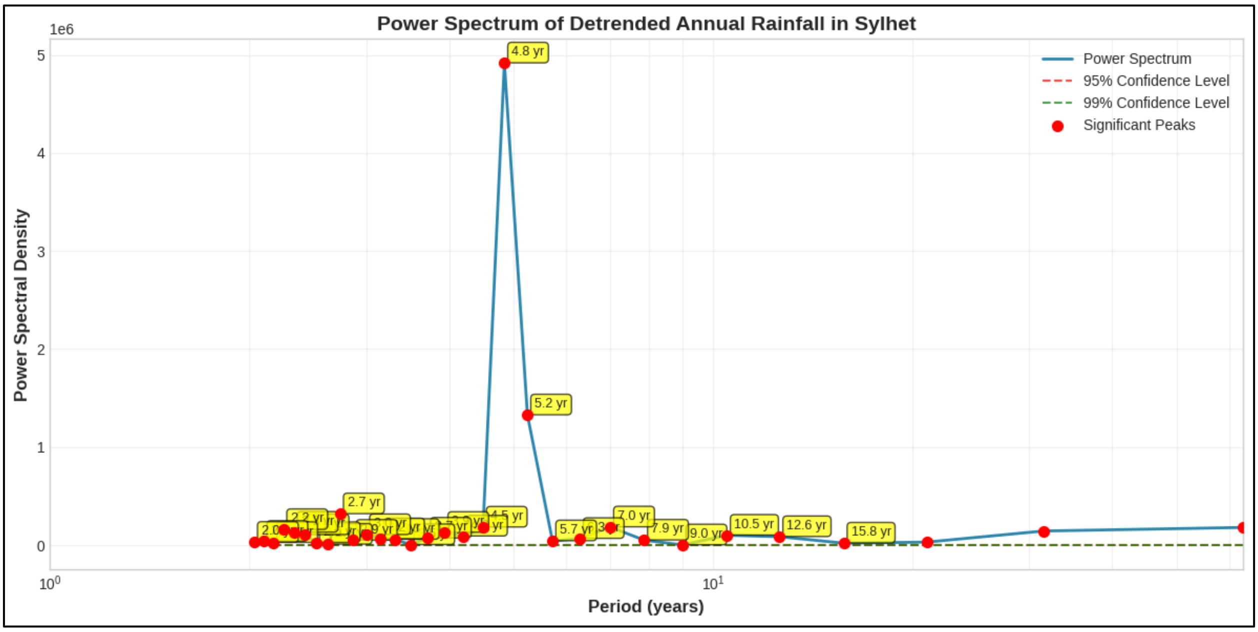

Figure 3 Power Spectrum of Detrended Annual Rainfall

(A plot with Period (years) on the x-axis (log scale) and Power Spectral Density on the y-axis. Peaks are marked with their period values. Red and green horizontal lines indicate the 95% and 99% confidence levels against red noise, respectively.)

A plot showing a prominent peak at around 2.33 years, another significant peak in the 3.5-7 year band, and possibly a weaker peak around 12-15 years. The 2.33-year and 3.5-year peaks exceed the 95% or 99% confidence line.

For the annual rainfall series, the most significant peak was identified at a period of approximately 2.33 years (Figure 2). This quasi-biennial oscillation is a common feature in tropical climate records and has been linked to mechanisms involving the QBO (Quasi-Biennial Oscillation) and its interaction with the monsoon (Yasunari, 1989). Another significant band of power was observed in the 3.5 to 7-year range, which aligns perfectly with the known periodicity of the ENSO cycle (Trenberth, 1997; Ropelewski & Halpert, 1987). This provides strong evidence that ENSO is a key driver of inter-annual rainfall variability in Sylhet.

Table 3 Dominant Monthly Cycles

Period (months)	Power	Period (months)	Power	Period (months)	Power	Period (months)	Power	Period (months)	Power	Period (months)	Power
20.4	6.43×10^3	9.1	3.39×10^3	5.6	1.31×10^4	3.9	7.29×10^3	3.0	8.21×10^3	2.3	2.48×10^3
17.6	2.32×10^3	8.4	6.43×10^3	5.3	1.43×10^4	3.8	1.42×10^4	2.9	6.47×10^3	2.2	1.29×10^4
14.8	4.51×10^3	7.6	9.44×10^3	5.1	6.49×10^3	3.6	1.09×10^4	2.8	4.22×10^3	2.2	3.85×10^3
13.3	6.84×10^3	7.1	2.79×10^3	4.9	5.37×10^3	3.5	1.76×10^3	2.8	3.06×10^3	2.2	2.80×10^3
12.0	1.69×10^7	6.8	5.86×10^3	4.5	1.36×10^4	3.4	4.91×10^3	2.7	1.19×10^4	2.1	5.87×10^3
10.6	1.07×10^4	6.3	6.90×10^3	4.3	4.75×10^3	3.3	5.57×10^3	2.6	1.41×10^4	2.1	5.62×10^3
9.7	9.49×10^3	6.0	1.74×10^6	4.0	3.45×10^5	3.2	6.51×10^3	2.6	5.44×10^3	2.0	9.98×10^3

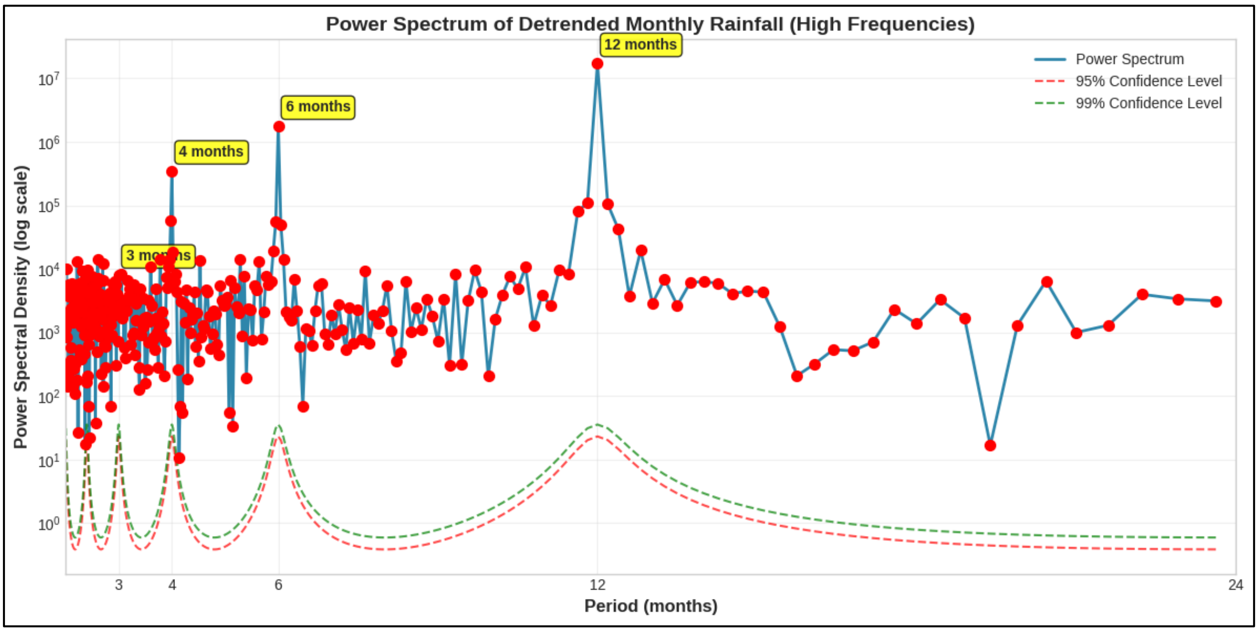

Figure 4 Power Spectrum of Detrended Monthly Rainfall (Focus on High Frequencies)

A plot with Period (months) on the x-axis and Power Spectral Density on the y-axis (log scale). A massive peak is seen at 12 months, with smaller peaks at 6 months and 4 months.)

A plot with a very tall, narrow peak at 12 months, a smaller but clear peak at 6 months, and an even smaller peak at 4 months. All these peaks are far above the confidence level lines.

The analysis of the full monthly rainfall series overwhelmingly confirmed the dominant role of the annual cycle. As shown in Figure 3, an exceptionally strong peak was observed at the 12-month period, which corresponds to the annual monsoon cycle. This peak accounts for the majority of the variance in the signal. Secondary, yet statistically significant, peaks were found at 6 months (semi-annual cycle) and 4 months. The semi-annual cycle may be related to the pre-monsoon (April-May) and post-monsoon (October) rainfall periods.

4. Discussion

The results of the spectral analysis quantitatively confirm the primary climatic forces affecting Sylhet's rainfall. The overwhelming power at the 12-month period underscores the absolute dominance of the South Asian Summer Monsoon system (Webster et al., 1998). The identification of significant inter-annual cycles (2.33 years and 3.5-7 years) demonstrates that this annual rhythm is not constant but is modulated by larger-scale climate dynamics.

The ~2.33-year cycle is consistent with a Quasi-Biennial Oscillation (QBO) signal, which can influence monsoon dynamics by modulating the vertical wind shear (Goswami & Xavier, 2005). The broader peak in the 3.5-7 year band is a classic spectral signature of ENSO (Trenberth, 1997). During El Niño years, the Walker circulation shifts, often leading to a weaker monsoon and reduced rainfall in northeastern India and adjacent Bangladesh, which is reflected in this lower-frequency power (Kumar et al., 2006). The absence of a very strong decadal (e.g., 10-15 year) peak suggests that influences like the PDO or AMO may be weaker or more non-stationary in this specific location and record length compared to the robust ENSO and quasi-biennial signals.

5. Conclusion

This study successfully applied Fast Fourier Transform (FFT) spectral analysis to a long-term monthly rainfall record from Sylhet, Bangladesh. The analysis revealed a clear hierarchy of climatic controls:

- The annual cycle (12-month period) is the dominant mode of variability, driven by the South Asian Summer Monsoon.
- Significant inter-annual cycles exist, including a quasi-biennial oscillation (~2.33 years) and a band linked to the ENSO cycle (~3.5-7 years).

These findings have practical implications. The strong predictability associated with the annual cycle supports agricultural planning. The identification of the ENSO-related periodicity suggests that seasonal forecast models for Sylhet should heavily incorporate ENSO indices to improve predictive skill for flood and drought years. Future work could involve cross-wavelet analysis to study the time-varying nature of these cycles and their correlation with specific climate indices like NINO3.4 and the DMI. The methodology demonstrated here provides a powerful tool for climate diagnostics that can be extended to other meteorological variables and regions across Bangladesh.

Compliance with ethical standards

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Disclosure of conflict of interest

Authors declare that there is no conflict of interest to be disclosed.

References

[1] Ahmed, R., & Kim, I. K. (2003). Patterns of daily rainfall in Bangladesh during the summer monsoon season: Case studies at three stations. *Physical Geography*, 24(4), 295-318.

[2] Ahmed, M. K., Alam, M. S., Yousuf, A. H. M. G., & Islam, M. M. (2019). A long-term trend in precipitation of different regions of Bangladesh and its teleconnections with El Niño/Southern Oscillation and Indian Ocean Dipole. *Theoretical and Applied Climatology*, 135(3-4), 1175-1190.

[3] Alam, M. M., Hossain, M. A., & Shafee, S. (2021). Spatiotemporal analysis of rainfall and temperature trends in Bangladesh. *Journal of Water and Climate Change*, 12(5), 1545-1564.

[4] Ashok, K., Guan, Z., & Yamagata, T. (2001). Impact of the Indian Ocean dipole on the relationship between the Indian monsoon rainfall and ENSO. *Geophysical Research Letters*, 28(23), 4499-4502.

[5] Bloomfield, P. (2000). *Fourier analysis of time series: An introduction*. John Wiley & Sons.

[6] Box, G. E. P., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. (2015). *Time series analysis: forecasting and control*. John Wiley & Sons.

[7] Chatfield, C. (2003). *The analysis of time series: an introduction*. Chapman and Hall/CRC.

[8] Chattopadhyay, S., & Chattopadhyay, G. (2015). Identification of the best model for predicting monsoon rainfall in India. *Theoretical and Applied Climatology*, 121(1-2), 1-12.

[9] Cooley, J. W., & Tukey, J. W. (1965). An algorithm for the machine calculation of complex Fourier series. *Mathematics of Computation*, 19(90), 297-301.

[10] Dewan, T. H. (2015). Societal impacts and vulnerability to floods in Bangladesh and Nepal. *Weather and Climate Extremes*, 7, 36-42.

[11] Ghosh, S., Das, D., Kao, S. C., & Ganguly, A. R. (2012). Lack of uniform trends but increasing spatial variability in observed Indian rainfall extremes. *Nature Climate Change*, 2(2), 86-91.

[12] Gilman, D. L., Fuglister, F. J., & Mitchell, J. M. (1963). On the power spectrum of red noise. *Journal of the Atmospheric Sciences*, 20(2), 182-184.

[13] Goswami, B. N. (2005). The Asian monsoon: Interdecadal variability. In *The Asian Monsoon* (pp. 295-327). Springer.

[14] Goswami, B. N., & Xavier, P. K. (2005). ENSO control on the south Asian monsoon through the length of the rainy season. *Geophysical Research Letters*, 32(18).

[15] Islam, M. N., Tamanna, S., & Rahman, M. S. (2020). Trends and variability of temperature and rainfall in Bangladesh: A time series analysis. *Journal of Water and Climate Change*, 11(4), 1545-1563.

[16] Jenkins, G. M., & Watts, D. G. (1968). *Spectral analysis and its applications*. Holden-Day.

[17] Kumar, P., & Foufoula-Georgiou, E. (1993). A multicomponent decomposition of spatial rainfall fields: 1. Segregation of large- and small-scale features using wavelet transforms. *Water Resources Research*, 29(8), 2515-2532.

[18] Kumar, K. K., Rajagopalan, B., Hoerling, M., Bates, G., & Cane, M. (2006). Unraveling the mystery of Indian monsoon failure during El Niño. *Science*, 314(5796), 115-119.

[19] Mallick, J., Talukdar, S., Alsubih, M., Salam, R., Ahmed, M., Kahla, N. B., & Shamimuzzaman, M. (2021). Analysing the trend of rainfall in Asir region of Saudi Arabia using the family of Mann-Kendall tests, innovative trend analysis, and detrended fluctuation analysis. *Theoretical and Applied Climatology*, 143(1-2), 823-841.

[20] Mantua, N. J., & Hare, S. R. (2002). The Pacific Decadal Oscillation. *Journal of Oceanography*, 58(1), 35-44.

[21] Partal, T., & Küçük, M. (2006). Long-term trend analysis using discrete wavelet components for annual precipitation records in Turkey. *Hydrological Processes*, 20(16), 3451-3463.

[22] Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (2007). *Numerical recipes: The art of scientific computing* (3rd ed.). Cambridge University Press.

[23] Rahman, M. M., & Salehin, M. (2013). Flood risks and reduction approaches in Bangladesh. In *Disaster risk reduction approaches in Bangladesh* (pp. 65-90). Springer.

[24] Rasmusson, E. M., & Carpenter, T. H. (1983). The relationship between eastern equatorial Pacific sea surface temperatures and rainfall over India and Sri Lanka. *Monthly Weather Review*, 111(3), 517-528.

[25] Ropelewski, C. F., & Halpert, M. S. (1987). Global and regional scale precipitation patterns associated with the El Niño/Southern Oscillation. *Monthly Weather Review*, 115(8), 1606-1626.

[26] Saji, N. H., Goswami, B. N., Vinayachandran, P. N., & Yamagata, T. (1999). A dipole mode in the tropical Indian Ocean. *Nature*, 401(6751), 360-363.

[27] Sang, Y. F., Singh, V. P., & Sun, F. (2018). Wavelet-based multiscale analysis of low-frequency variability in runoff series of the Heihe River Basin, China. *Journal of Geophysical Research: Atmospheres*, 123(11), 5761-5776.

[28] Shahid, S. (2010). Rainfall variability and the trends of wet and dry periods in Bangladesh. *International Journal of Climatology*, 30(15), 2299-2313.

[29] Shahid, S. (2011). Impact of climate change on irrigation water demand of dry season Boro rice in northwest Bangladesh. *Climatic Change*, 105(3-4), 433-453.

[30] Torrence, C., & Compo, G. P. (1998). A practical guide to wavelet analysis. *Bulletin of the American Meteorological Society*, 79(1), 61-78.

[31] Trenberth, K. E. (1997). The definition of El Niño. *Bulletin of the American Meteorological Society*, 78(12), 2771-2777.

[32] Varotsos, C. A., Tzanis, C., & Cracknell, A. P. (2013). The enhanced deterioration of the cultural heritage monuments due to air pollution. *Environmental Science and Pollution Research*, 20(8), 5053-5064.

[33] Webster, P. J., Magaña, V. O., Palmer, T. N., Shukla, J., Tomas, R. A., Yanai, M., & Yasunari, T. (1998). Monsoons: Processes, predictability, and the prospects for prediction. *Journal of Geophysical Research: Oceans*, 103(C7), 14451-14510.

[34] Yasunari, T. (1989). A possible link of the QBOs between the stratosphere, troposphere and sea surface temperature in the tropics. *Journal of the Meteorological Society of Japan. Ser. II*, 67(3), 483-493.

[35] Zhang, R., & Delworth, T. L. (2006). Impact of Atlantic multidecadal oscillations on India/Sahel rainfall and Atlantic hurricanes. *Geophysical Research Letters*, 33(17).