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Optimization of energy losses in a certain class of woodworking machines - analytical and numerical solutions

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Abstract

In this paper, kinetic energy losses in big band saw machines are investigated. These losses are caused by the linear and angular inaccuracies with which the leading wheels are made. Expressions are obtained for calculating the kinetic energy of the mechanical system in the ideal and real cases when the two wheels are made without and with linear and angular inaccuracies. Using these expressions, an expression for calculating the kinetic energy loss of the mechanical system is obtained. Various optimization solutions have been proposed to calculate the optimal values of the parameters that take into account the linear and angular inaccuracies. At these values, kinetic energy losses in big band saw machines will be minimal. The proposed study can be used to minimize energy losses in other classes of woodworking machines.

Keywords: Woodworking Machines; Kinetic Energy Losses; Optimization Solutions; Linear and Angular Inaccuracies

1. Introduction

In this study, the energy losses in big band saw machines are analysed. These machines are widely used in the woodworking industry because of their high productivity. Their work, however, is characterized by high-energy consumption, which is largely because main links of the machine are very large. For example, the diameters of the two leading wheels may vary from 1100 to 3200 *mm*, and the width of the band saw blade varies from 140 to 350 *mm*. Due to this fact, both wheels are made with corresponding inaccuracies that cannot be avoided.

Scientific studies related to energy efficiency of different classes of woodworking machinery are published in the technical literature [1-5].

The purpose of the proposed study is to determine the energy losses of the mechanical system as a result of these inaccuracies. In order to achieve this purpose, the following basic tasks must be solved:

- Obtaining expressions for calculating the kinetic energy of the mechanical system in the absence of linear and angular inaccuracies;
- Obtaining expressions for calculating the kinetic energy of the mechanical system in the presence of linear and angular inaccuracies;
- Obtaining an expression to calculate the kinetic energy loss of the mechanical system in the real case.

Another purpose of this study is to present various optimization solutions that can reduce the actual energy consumption of the band saw machines in operation mode. In this case, it is necessary to use optimization procedures, as well as to compile the appropriate objective functions. These functions are used by the optimization procedures to

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calculate the optimal parameter values. In this way, it can be ensured that the band saw machines will operate in optimal mode with minimum energy losses.

2. Expose

In this part of the study, the scheme of the cutting mechanism of the band saw machine is used. The dynamic models in the absence and presence of linear and angular inaccuracies are also proposed. They are used to fulfill the purposes of the study as well as to solve the written tasks.

2.1. Scheme of the cutting mechanism

The leading wheel of the big band saw machines can be mounted on the upper shaft in two ways. These two cases are written as case (a) and case (b) and are explained below [6-9].

- Case (a)-leading wheel is mounted at the end of the shaft,
- Case (b)-leading wheel is mounted between the two bearing supports.

The scheme of the cutting mechanism is shown in Fig. 1.

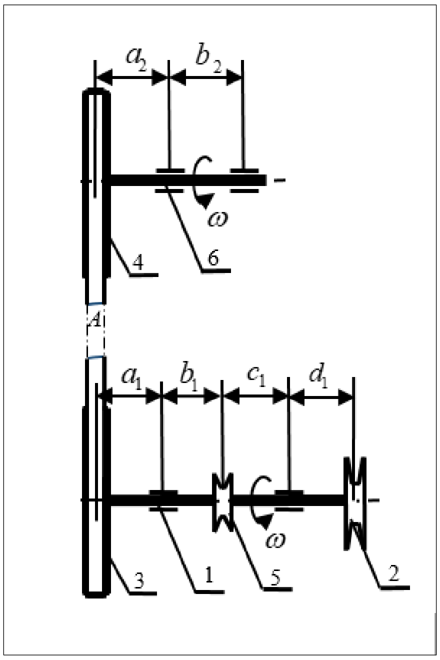
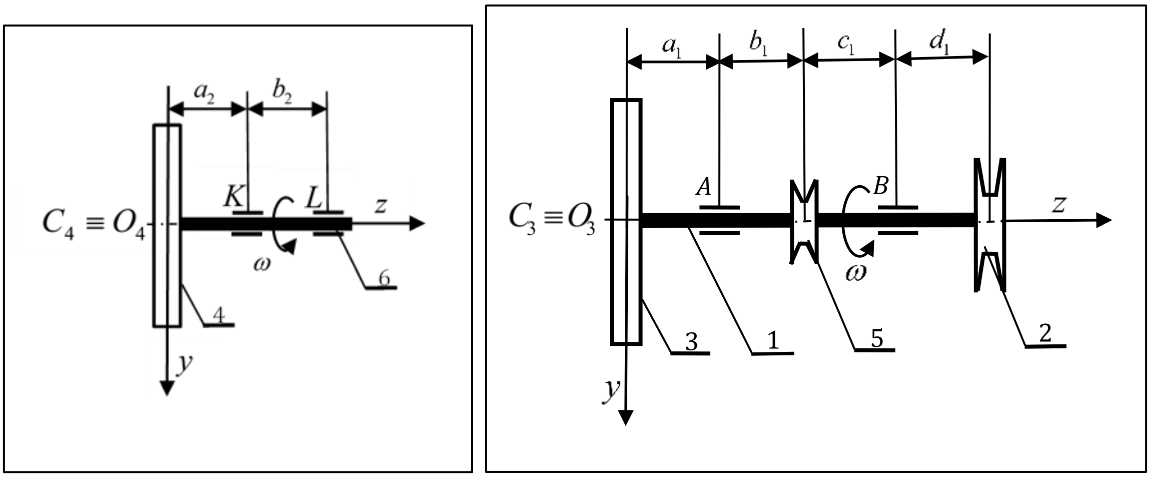


Figure 1 Cutting mechanism-case(a)

The following symbols are defined: 1-main shaft, 2 and 5-belt pulleys, 3 and 4-leading wheels, A-band saw blade, 6-upper shaft.

2.2. Kinetic energy of the mechanical system without linear and angular inaccuracies - ideal case

We use the dynamic models of the leading wheels mounted on the upper shaft and on the main shaft of the band saw machine to calculate the kinetic energy. These models are shown in Figs. 2(a) and 2(b).



(a) (b)

Figure 2 Dynamic models in the absence of linear and angular inaccuracies

In this case, two coordinate systems O_4xyz and O_3xyz are used. It should be noted that for these coordinate systems the *x*-axis is directed towards us. The shafts, belt pulleys and leading wheels rotate at a constant angular velocity ω.

2.2.1. Kinetic energy of the mechanical system shown in figure 2(a)-ideal case

The kinetic energy of the mechanical system can be calculated from the following dependence [10-12]:

$$T_{pa} = \frac{1}{2} J_{pa}\omega^2, \tag{1}$$

where J_{pa} is the mass moments of inertia of the mechanical system with respect to the axis of rotation.

$$J_{pa} = J_{4a} + J_{6a}, \tag{2}$$

where J_{4a} and J_{6a} are the mass moments of inertia of the leading wheel 4 and of the upper shaft 6 relative to the z axis.

2.2.2. Kinetic energy of the mechanical system shown in figure 2(b)-ideal case

The kinetic energy in this case can be calculated by the following expression.

$$T_p = \frac{1}{2} J_p\omega^2, \tag{3}$$

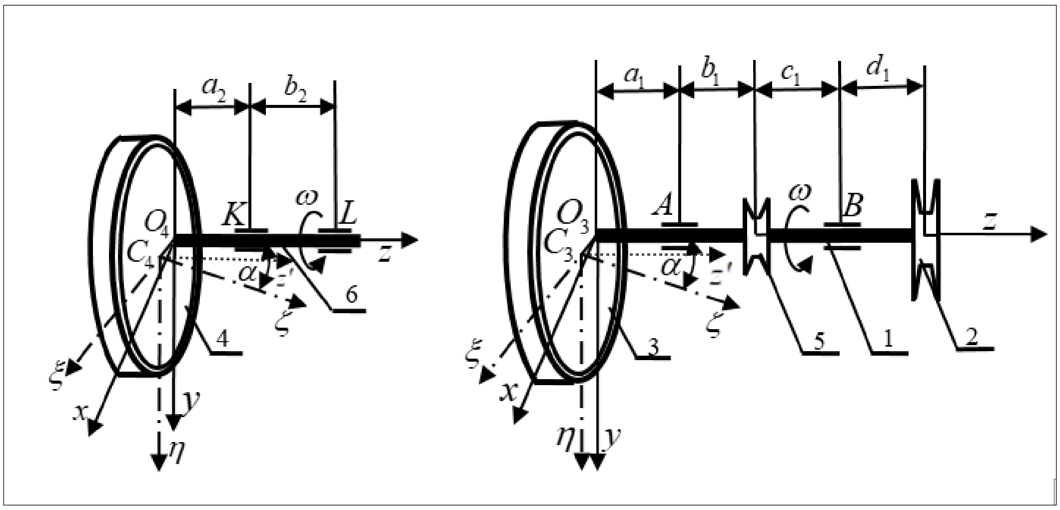
where J_p is the mass moment of inertia of the system with respect to the axis of rotation, i.e. toward the axis z. We can calculate it from the following expression:

$$J_p = J_2 + J_3 + J_5 + J_{sh}. \tag{4}$$

In the above expression, J_2 and J_5 are the mass moments of inertia of the belt pulleys 2 and 5, J_3 is the mass moment of inertia of the leading wheel 3 and J_{sh} is the mass moment of inertia of the main shaft with respect to the axis of rotation, ω is the angular velocity of the mechanical system toward the same axis.

2.3. Kinetic energy of the mechanical system in the presence of linear and angular inaccuracies - real case

In this case, we use the dynamic models shown in Figs. 3(*a*) and 3(*b*). The leading wheels 3 and 4 are made with the corresponding linear and angular inaccuracies. These deviations are shown in the same figures and are denoted as linear inaccuracies $e=O_jC_j$, ($j=3,4$), where C_j are the centres of masses of the discs 3 and 4 and angular inaccuracies α.



(a) (b)

Figure 3 Dynamic models in the presence of linear and angular inaccuracies

2.3.1. Kinetic energy of the mechanical system shown in figure 3(a) - real case

The dynamic model shown in Fig. 3(a) includes the upper leading wheel 4 and the upper shaft 6. We use the expression for calculating the kinetic energy of the mechanical system in its final form. This expression was obtained by the author in his previous work [8].

$$T_{ra} = \frac{1}{2}\left(J_{ra} + m_4 Z_{21}^2\right)\omega^2, \tag{5}$$

where J_{ra} in the expression (5) is the mass moment of inertia of the mechanical system and can be calculated from the following expression,

$$J_{ra} = J_{4a}^{'} + J_{6a}. \tag{6}$$

In the above expression, $J_{4a}^{'}$ is the mass moment of inertia of the leading wheel relative to the axis of rotation. This moment can be calculated from a known formula in the technical literature [10].

$$J_{4a}^{'} = J_{\zeta}\cos^2\alpha + J_{\xi}\sin^2\alpha + m_4 e^2\cos^2\alpha, \tag{7}$$

where J_{ζ} and J_{ξ} are the mass moments of inertia of the leading wheel 4 with respect to the principal axes of inertia ζ and ξ, m_4 is the mass of leading wheel 4.

The function Z_{21} is calculated from the expression (8),

$$Z_{21} = \overline{K}_1 + \overline{M}_1. \tag{8}$$

In this expression $\overline{K}_1$ and $\overline{M}_1$ are constants of integration. They can be calculated in the following way:

$$\overline{K}_1 = \frac{\Delta_{\overline{K}_1}}{\Delta_M},\ \overline{M}_1 = \frac{\Delta_{\overline{M}_1}}{\Delta_M}, \tag{9}$$

where $\Delta_M = \det M$ is the determinant of the matrix **M.** This determinant is non-zero, because otherwise the mechanical system will operate in resonance mode.

M= (10)

$$\begin{bmatrix}
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\cos\omega_0 a_2 & \sin\omega_0 a_2 & ch\omega_0 a_2 & sh\omega_0 a_2 & 0 & 0 & 0 & 0 \\
-\sin\omega_0 a_2 & \cos\omega_0 a_2 & sh\omega_0 a_2 & ch\omega_0 a_2 & \sin\omega_0 a_2 & -\cos\omega_0 a_2 & -sh\omega_0 a_2 & -ch\omega_0 a_2 \\
-\sin\omega_0 a_2 & \cos\omega_0 a_2 & -sh\omega_0 a_2 & -ch\omega_0 a_2 & \sin\omega_0 a_2 & -\cos\omega_0 a_2 & sh\omega_0 a_2 & ch\omega_0 a_2 \\
0 & 0 & 0 & 0 & \cos\omega_0 a_2 & \sin\omega_0 a_2 & ch\omega_0 a_2 & sh\omega_0 a_2 \\
0 & 0 & 0 & 0 & \cos\omega_0(a_2+b_2) & \sin\omega_0(a_2+b_2) & ch\omega_0(a_2+b_2) & sh\omega_0(a_2+b_2) \\
0 & 0 & 0 & 0 & -\sin\omega_0(a_2+b_2) & \cos\omega_0(a_2+b_2) & -sh\omega_0(a_2+b_2) & -ch\omega_0(a_2+b_2)
\end{bmatrix}$$

The other determinants in expressions (9) are determinants of the matrices formed by the matrix **M**, in which the corresponding columns are replaced by the column-vector **m** as follows:

$\Delta_{\overline{K}_1}$ - the first column of the matrix **M**,

(11)

$\Delta_{\overline{M}_1}$ - the third column of the matrix **M.**

The column-vector **m** is recorded below in expression (12),

$$\mathbf{m} = \left[\frac{m_4\omega^2 e\cos\alpha}{EJ\omega_0^3} \quad \frac{M_{bm}}{EJ\omega_0^2} \quad 0 \quad 0 \quad \frac{K_{bm}}{EJ\omega_0^3} \quad 0 \quad 0 \quad \frac{L_{bm}}{EJ\omega_0^3} \right]^T. \tag{12}$$

The determinant $\Delta_{\overline{K}_1}$ has the form written below and can be calculated from this expression.

$\Delta_{\overline{K}_1}$ = (13)

$$\begin{vmatrix}
\frac{m_4\omega^2 e\cos\alpha}{EJ\omega_0^3} & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
\frac{M_{bm}}{EJ\omega_0^2} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \sin\omega_0 a_2 & ch\omega_0 a_2 & sh\omega_0 a_2 & 0 & 0 & 0 & 0 \\
0 & \cos\omega_0 a_2 & sh\omega_0 a_2 & ch\omega_0 a_2 & \sin\omega_0 a_2 & -\cos\omega_0 a_2 & -sh\omega_0 a_2 & -ch\omega_0 a_2 \\
\frac{K_{bm}}{EJ\omega_0^3} & \cos\omega_0 a_2 & -sh\omega_0 a_2 & -ch\omega_0 a_2 & \sin\omega_0 a_2 & -\cos\omega_0 a_2 & sh\omega_0 a_2 & ch\omega_0 a_2 \\
0 & 0 & 0 & 0 & \cos\omega_0 a_2 & \sin\omega_0 a_2 & ch\omega_0 a_2 & sh\omega_0 a_2 \\
0 & 0 & 0 & 0 & \cos\omega_0 (a_2+b_2) & \sin\omega_0 (a_2+b_2) & ch\omega_0 (a_2+b_2) & sh\omega_0 (a_2+b_2) \\
\frac{L_{bm}}{EJ\omega_0^3} & 0 & 0 & 0 & -\sin\omega_0 (a_2+b_2) & \cos\omega_0 (a_2+b_2) & -sh\omega_0 (a_2+b_2) & -ch\omega_0 (a_2+b_2)
\end{vmatrix}$$

The amplitudes K_{bm}, L_{bm} and M_{bm} are calculated from the following dependencies:

$$K_{bm} = -\frac{1}{2b_2}\left(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\right)e\cos\alpha - \frac{\omega^2}{b_2}\left[\left(a_2 + b_2 + e\sin\alpha\right)m_4 e\cos\alpha - \frac{1}{2}\left(J_\xi - J_\zeta\right)\sin 2\alpha\right],$$

$$L_{bm} = \frac{1}{2b_2}\left(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\right)e\cos\alpha + \frac{\omega^2}{b_2}\left[\left(a_2 + e\sin\alpha\right)m_4 e\cos\alpha - \frac{1}{2}\left(J_\xi - J_\zeta\right)\sin 2\alpha\right], \tag{14}$$

$$M_{bm} = \frac{1}{2}\left[\left(\frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma\right)e\cos\alpha - \omega^2\left(J_\xi - J_\zeta - m_4 e^2\right)\sin 2\alpha\right].$$

In the above dependencies ω_0 is parameter, $(\omega_0^4 = A_{ms}\rho\omega^2 / EJ)$, [13, 14], E is the modulus of elasticity, $J = J_x = J_y$ is the axial moment of inertia, A_{ms} is the cross-sectional area of the shaft, ρ is the density of the material, V is the cutting speed, H is the thickness of the workpiece, u is the feeding speed, b is the width of the cutter. m is a coefficient that changes within the following limits $0 \le m \le 1$. $K_{\Delta(\lambda)}$ is the specific work of the cutting. It is determined from the expressions known in technical literature. [6, 15, 16].

2.3.2. Kinetic energy of the mechanical system shown in figure 3(b) - real case

The dynamic model shown in Fig. 3(b) includes the lower leading wheel 3, the main shaft 1 and the belt pulleys 2 and 5. In this case, we use the following expression to calculate the kinetic energy of the mechanical system [8],

$$T_r = \frac{1}{2}\left(J_r + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right)\omega^2. \tag{15}$$

In this expression m_3, m_5 and m_2 are the masses of the three disks.

J_r is the mass moment of inertia of the system in the presence of linear and angular inaccuracies, i.e., in the real case. This moment can be calculated from the following expression:

$$J_r = J_2 + J_3^{'} + J_5 + J_1 . \tag{16}$$

The mass moment of inertia of the leading wheel with respect to the axis of rotation is denoted by $J_3^{'}$ and can be calculated using expression (17).

$$J_3^{'} = J_\zeta \cos^2\alpha + J_\xi \sin^2\alpha + m_3 e^2 \cos^2\alpha \, , \tag{17}$$

where J_ζ and J_ξ are the mass moments of inertia of the leading wheel 3 with respect to the principal axes of inertia ζ and ξ.

The functions Z_{2i}, $(i = 1 \div 3)$ are calculated from the dependencies written below.

$$\begin{aligned} Z_{21} &= \left(\overline{R}_1 + \overline{V}_1\right), \\ Z_{22} &= \left(\overline{R}_2 \cos\omega_0 z_2 + \overline{S}_2 \sin\omega_0 z_2 + \overline{V}_2 ch\omega_0 z_2 + \overline{W}_2 sh\omega_0 z_2\right), \\ Z_{23} &= \left(\overline{R}_3 \cos\omega_0 z_3 + \overline{S}_3 \sin\omega_0 z_3 + \overline{V}_3 ch\omega_0 z_3 + \overline{W}_3 sh\omega_0 z_3\right), \end{aligned} \tag{18}$$

where the coordinates z_i $(i = 1 \div 3)$ have the following values: $z_1 = 0$, $z_2 = a_1 + b_1$, $z_3 = a_1 + b_1 + c_1 + d_1 = l$.

$\overline{R}_i, \overline{S}_i, \overline{V}_i, \overline{W}_i,$ $(i = 1 \div 3)$ are the constants of integration. They can be calculated from the following dependencies:

$$\overline{R}_i = \frac{\Delta_{\overline{R}_i}}{\Delta_B}, \quad \overline{S}_i = \frac{\Delta_{\overline{S}_i}}{\Delta_B}, \quad \overline{V}_i = \frac{\Delta_{\overline{V}_i}}{\Delta_B}, \quad \overline{W}_i = \frac{\Delta_{\overline{W}_i}}{\Delta_B}, \quad (i = 1, 2, 3), \tag{19}$$

where $\Delta_B = \det B$ is the determinant of the matrix **B**, which has the following form [8, 15]:

B= Columns 1 through 7 (20)

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \cos\omega_0 a_1 & \sin\omega_0 a_1 & ch\omega_0 a_1 & sh\omega_0 a_1 & 0 & 0 & 0 \\ -\sin\omega_0 a_1 & \cos\omega_0 a_1 & sh\omega_0 a_1 & ch\omega_0 a_1 & \sin\omega_0 a_1 & -\cos\omega_0 a_1 & -sh\omega_0 a_1 \\ -\sin\omega_0 a_1 & \cos\omega_0 a_1 & -sh\omega_0 a_1 & -ch\omega_0 a_1 & \sin\omega_0 a_1 & -\cos\omega_0 a_1 & sh\omega_0 a_1 \\ 0 & 0 & 0 & 0 & \cos\omega_0 a_1 & \sin\omega_0 a_1 & ch\omega_0 a_1 \\ 0 & 0 & 0 & 0 & \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & ch\omega_0(a_1+b_1+c_1) \\ 0 & 0 & 0 & 0 & -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & -sh\omega_0(a_1+b_1+c_1) \\ 0 & 0 & 0 & 0 & -\sin\omega_0(a_1+b_1+c_1) & \cos\omega_0(a_1+b_1+c_1) & sh\omega_0(a_1+b_1+c_1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Columns 8 through 12

$$\left.\begin{matrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-ch\omega_0 a_1 & 0 & 0 & 0 & 0 \\
ch\omega_0 a_1 & 0 & 0 & 0 & 0 \\
sh\omega_0 a_1 & 0 & 0 & 0 & 0 \\
sh\omega_0(a_1+b_1+c_1) & 0 & 0 & 0 & 0 \\
-ch\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & -\cos\omega_0(a_1+b_1+c_1) & sh\omega_0(a_1+b_1+c_1) & ch\omega_0(a_1+b_1+c_1) \\
ch\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & -\cos\omega_0(a_1+b_1+c_1) & -sh\omega_0(a_1+b_1+c_1) & -ch\omega_0(a_1+b_1+c_1) \\
0 & \cos\omega_0(a_1+b_1+c_1) & \sin\omega_0(a_1+b_1+c_1) & ch\omega_0(a_1+b_1+c_1) & sh\omega_0(a_1+b_1+c_1) \\
0 & -\cos\omega_0(a_1+b_1+c_1+d_1) & -\sin\omega_0(a_1+b_1+c_1+d_1) & ch\omega_0(a_1+b_1+c_1+d_1) & sh\omega_0(a_1+b_1+c_1+d_1) \\
0 & \sin\omega_0(a_1+b_1+c_1+d_1) & -\cos\omega_0(a_1+b_1+c_1+d_1) & sh\omega_0(a_1+b_1+c_1+d_1) & ch\omega_0(a_1+b_1+c_1+d_1)
\end{matrix}\right]$$

The other determinants in (19) are determinants of matrices that are formed by matrix **B**. In this matrix, certain columns are replaced by the column-vector **b** as follows:

$\Delta_{\bar{R}_i}$ $(i = 1 \div 3)$ - the first, the fifth, or the ninth column,

$\Delta_{\bar{S}_i}$ $(i = 1 \div 3)$ - the second, the sixth or the tenth column, (21)

$\Delta_{\bar{V}_i}$ $(i = 1 \div 3)$ - the third, the seventh or the eleventh column,

$\Delta_{\bar{W}_i}$ $(i = 1 \div 3)$ - the fourth, the eighth or the twelfth column.

The column-vector **b** is written below,

$$\mathbf{b} = \begin{bmatrix} \dfrac{m_3 e\omega^2 \cos\alpha}{EJ\omega_0^3} & \dfrac{M_{bm}}{EJ\omega_0^2} & 0 & 0 & \dfrac{A_{bm}}{EJ\omega_0^3} & 0 & 0 & \dfrac{B_{bm}}{EJ\omega_0^3} & 0 & 0 & 0 & 0 \end{bmatrix}^T . \quad (22)$$

The symbols E, J and ω_0 are explained in formula (14). A_{bm}, B_{bm} and M_{bm} can be calculated from the following expressions:

$$A_{bm} = \frac{1}{2(b_1+c_1)}\left[\omega^2(J_\xi - J_\varsigma)\sin 2\alpha - 2\omega^2 m_3(a_1+b_1+c_1+e\sin\alpha)e\cos\alpha - \left(R_b^n + R_\Sigma\right)e\cos\alpha\right],$$

$$B_{bm} = \frac{1}{2(b_1+c_1)}\left[\left(R_b^n + R_\Sigma\right)e\cos\alpha + 2\omega^2 m_3(a_1+e\sin\alpha)e\cos\alpha - \omega^2(J_\xi - J_\varsigma)\sin 2\alpha\right], \quad (23)$$

$$M_{bm} = \frac{1}{2}\left[\left(R_b^n + R_\Sigma\right)e\cos\alpha - \omega^2\left(J_\xi - J_\varsigma - m_3 e^2\right)\sin 2\alpha\right],$$

where R_b^n is the normal force loading the workpiece. This force can be calculated from expressions that are used in various sources [6, 15, 16]. R_Σ is the total resistance force between the workpiece and the band saw machine. This force is calculated for each individual case.

2.4. Loss of kinetic energy of the mechanical system

2.4.1. Loss of kinetic energy of the mechanical system consisting of the upper shaft and the upper leading wheel

The loss of kinetic energy can be calculated from the following expression:

$$_{\Delta}T_a = T_{ra} - T_{pa}. \tag{24}$$

We replace T_{ra} and T_{pa} with their equals (5) and (1) and obtain the following expression:

$$_{\Delta}T_a = \frac{1}{2}\left(J_{ra} + m_4 Z_{21}^2\right)\omega^2 - \frac{1}{2} J_{pa}\omega^2. \tag{25}$$

From this expression we obtain the expression for calculating the kinetic energy losses in a final form,

$$_{\Delta}T_a = \frac{1}{2}\left[\left(J_{\xi} - J_{\zeta}\right)\sin^2\alpha + m_4 e^2 \cos^2\alpha + m_4\left(\overline{K}_1 + \overline{M}_1\right)^2\right]\omega^2. \tag{26}$$

2.4.2. Loss of kinetic energy of the mechanical system consisting of the main shaft, the lower leading wheel and the belt pulleys

The loss of kinetic energy can be calculated from the following expression:

$$_{\Delta}T = T_r - T_p. \tag{27}$$

We replace T_r and T_p with their equal expressions written in (15) and (3) and get the following expression:

$$_{\Delta}T = \frac{1}{2}\left(J_r + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right)\omega^2 - \frac{1}{2} J_p\omega^2. \tag{28}$$

After converting the above expression, we obtain the following dependence for calculating the kinetic energy loss of the mechanical system,

$$_{\Delta}T = \frac{1}{2}\left[\left(J_{\xi} - J_{\zeta}\right)\sin^2\alpha + m_3 e^2 \cos^2\alpha + m_3 Z_{21}^2 + m_5 Z_{22}^2 + m_2 Z_{23}^2\right]\omega^2. \tag{29}$$

2.4.3. Complete loss of kinetic energy of the mechanical system

The complete kinetic energy loss of the mechanical system is the sum of the energy loss of the upper shaft and upper leading wheel and the energy loss of the main shaft, lower leading wheel, and two belt pulleys. This loss can be calculated from the following expression:

$$_{\Delta}T_{\Sigma} = {_{\Delta}T_a} + {_{\Delta}T}. \tag{30}$$

We replace $_{\Delta}T_a$ and $_{\Delta}T$ with their equals and after appropriate transformations we get the expression for the complete loss of kinetic energy in its final form.

$$
{}_{\Delta}T_{\Sigma} = \frac{1}{2}\begin{bmatrix} \left(J_{\xi} - J_{\zeta}\right)\sin^2\alpha + m_4 e^2\cos^2\alpha + m_4\left(\overline{K}_1 + \overline{M}_1\right)^2 + \\ \left(J_{\xi} - J_{\zeta}\right)\sin^2\alpha + m_3 e^2\cos^2\alpha + m_3\left(\overline{R}_1 + \overline{V}_1\right)^2 + \\ m_5\left(\overline{R}_2\cos\omega_0 z_2 + \overline{S}_2\sin\omega_0 z_2 + \overline{V}_2 ch\omega_0 z_2 + \overline{W}_2 sh\omega_0 z_2\right)^2 + \\ m_2\left(\overline{R}_3\cos\omega_0 z_3 + \overline{S}_3\sin\omega_0 z_3 + \overline{V}_3 ch\omega_0 z_3 + \overline{W}_3 sh\omega_0 z_3\right)^2 \end{bmatrix}\omega^2 \qquad (31)
$$

This expression allows to calculate the loss of kinetic energy at different values of the linear and angular inaccuracies, as well as different values of the masses and of the geometric and kinematic characteristics of the mechanical system.

2.5. Kinetic energy losses in big band saw machines – optimization

2.5.1. Optimization solutions reducing energy losses

As can be seen from the expression (31), the loss of kinetic energy of big band saw machines depends on different parameters. The purpose of this part of the study is to propose optimization solutions that minimize these losses. We use optimization procedure with constraints - *fmincon* [17, 18]. This procedure looks for the minimum of a multidimensional function with constraints. In this case, the interval of constraints in which the parameters e and α change must be determined. These constraints are expressed by the following inequalities: lb ≤ [e,α] ≤ ub, where lb and ub are the lower and upper bounds. It is also necessary to determine the initial approximation x0 around which the function has a local minimum. If the initial approximation is inappropriate, the procedure selects a new initial approximation. The way in which the optimization procedure is applied is shown below. In order to start the procedure, it is necessary to create an *objective function*, which uses Eq. (31).

```
function F_def

x0 = [Initial Approximation];

lb = [Lower Bounds];

ub = [Upper Bounds];

ops = optimset('LargeScale','off');

[x,fval,exitflag,output]= fmincon(@def1,x0,A,b,Aeq,beq,lb,ub,nonlcon,ops),

function F1 = def1(x)
```

objective function

F_1 = [expression for calculating the loss of kinetic energy-see Eq. (31)].

2.5.2. Optimization procedure – application

Due to the large dimensions of the leading wheels, they are always produced with corresponding inaccuracies. The magnitudes of these inaccuracies depend on many factors, such as different manufacturing technologies, different assembly technologies, different operating conditions, and more. For this reason, we use the allowable technological boundaries in which the parameters e and α are changed, i.e. $e_{\min} \le e \le e_{\max}$, $\alpha_{\min} \le \alpha \le \alpha_{\max}$. These boundaries determine the zone in which the big band saw machines can operate. This zone is called the *working zone*. The area under the working zone is called the *initial zone* and the area above the working zone is called the *final zone*. We can apply the optimization procedures for the three zones. However, this is not always necessary. The most important area for which we are looking for an optimization solution is the working zone. We can obtain the optimal values of the parameters e and α so that the loss of kinetic energy has a minimum value. For this purpose, we must use the appropriate input data.

We can calculate the values for the change of kinetic energy using the obtained expression (31). In this case, the parameter e changes from 0 to 0.0015 with step 0.0003 and the parameter α changes from 0 to 0.026175 with step 0.001745. The obtained values are presented below as a matrix.

$$_{\Delta}T_{\Sigma} = 1.0\mathrm{e}+003 *$$

$$\begin{bmatrix} 0.0000 & 0.0181 & 0.0724 & 0.1630 & 0.2897 & 0.4526 & 0.6517 & 0.8870 \\ 0.0030 & 0.0359 & 0.1049 & 0.2102 & 0.3517 & 0.5293 & 0.7432 & 0.9932 \\ 0.0121 & 0.0597 & 0.1435 & 0.2635 & 0.4197 & 0.6121 & 0.8407 & 1.1055 \\ 0.0272 & 0.0896 & 0.1881 & 0.3228 & 0.4938 & 0.7009 & 0.9443 & 1.2238 \\ 0.0484 & 0.1255 & 0.2388 & 0.3883 & 0.5739 & 0.7958 & 1.0539 & 1.3481 \\ 0.0757 & 0.1675 & 0.2955 & 0.4597 & 0.6601 & 0.8968 & 1.1696 & 1.4785 \\ 1.1585 & 1.4661 & 1.8099 & 2.1898 & 2.6058 & 3.0578 & 3.5460 & 4.0701 \\ 1.2794 & 1.6018 & 1.9603 & 2.3549 & 2.7856 & 3.2524 & 3.7552 & 4.2940 \\ 1.4064 & 1.7435 & 2.1167 & 2.5260 & 2.9714 & 3.4529 & 3.9705 & 4.5240 \\ 1.5394 & 1.8912 & 2.2792 & 2.7032 & 3.1634 & 3.6596 & 4.1918 & 4.7601 \\ 1.6785 & 2.0450 & 2.4477 & 2.8865 & 3.3613 & 3.8723 & 4.4192 & 5.0022 \\ 1.8236 & 2.2049 & 2.6223 & 3.0758 & 3.5654 & 4.0910 & 4.6527 & 5.2504 \end{bmatrix}$$

As can be seen from the above matrix, the loss of kinetic energy changes from zero to 5250.4 [J] when changing the values of the two parameters. To find optimization solutions for the three zones where the dynamic system will operate in the optimal mode, we use the optimization procedure *fmincon*. For these zones, we define the lower bound lb, the upper bound ub, as well as the initial approximation x0.

Initial Zone

```
function F_def

x0=[0.00075 0.0026175];

lb=[0.0003 0];

ub=[0.0012 0.005235];

ops=optimset('LargeScale','off');

[x,fval,exitflag,output]=fmincon(@def1,x0,[],[],[],[],lb,ub,[],ops)

function F1=def1(x)
```

objective function

F_1=[see expressions for calculating the loss of kinetic energy (31)]

The following results are obtained after startup of the procedure.

e=1.0e - 003*0.3000 [m], α=0,

${}_{\Delta}\mathrm{T}_{\Sigma\,\min} = 3.0272\,[\mathrm{J}]$.

exitflag =1

output =

algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'

Ideal case: $e = 0, \quad \alpha = 0, \quad {}_{\Delta}\mathrm{T}_{\Sigma\min} = 0.$

Working Zone

```
function F_def

x0=[0.00075 0.0130875];

lb=[0.0003 0.005235];

ub=[0.0012 0.02094];

ops=optimset('LargeScale','off');

[x,fval,exitflag,output]=fmincon(@def1,x0,[],[],[],[],lb,ub,[],ops)

function F1=def1(x)
```

objective function

F_1=[see expressions for calculating the loss of kinetic energy (31)]

The following results are obtained after startup of the procedure.

e=0.0003 [m], α = 0.0052 [rad],

${}_{\Delta}\mathrm{T}_{\Sigma\,\min} = 210.1979\,[\mathrm{J}]$.

exitflag = 1

output =

algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'

Final Zone

```
function F_def

x0=[0.00075 0.023558];

lb=[0.0003 0.02094];

ub=[0.0012 0.026175];

ops=optimset('LargeScale','off');

[x,fval,exitflag,output]=fmincon(@def1,x0,[],[],[],[],lb,ub,[],ops)

function F1=def1(x)
```

objective function

F_1=[see expressions for calculating the loss of kinetic energy (31)]

.The following results are obtained after startup of the procedure.

e=1.0e - 003*0.3000 [m], α=0.0209 [rad],

$${}_{\Delta}\mathrm{T}_{\Sigma\min} = 2.7856\mathrm{e} + 003[\mathrm{J}]$$

exitflag =1

output =

algorithm: 'medium-scale: SQP, Quasi-Newton, line-search'

The worst case:

$$e = 0.0015[m], \quad \alpha = 0.026175[rad],$$

$${}_{\Delta}\mathrm{T}_{\Sigma\min} = 5.2504\mathrm{e} + 003[\mathrm{J}].$$

2.6. Summary analysis of the formulated purpose and the main tasks for implementation

In this research, optimization solutions are proposed for determining the minimum value of kinetic energy loss ${}_{\Delta}T_{\Sigma}$. As can be seen from the numerical values written as matrices, the loss of kinetic energy depends on the two parameters e and α and increases with increasing values of these parameters. For example, ${}_{\Delta}T_{\Sigma}$ changes in the following limits $0 \le {}_{\Delta}T_{\Sigma} \le 5250.4[J]$.

The most important area in which the band saw machine works is the working zone. The minimum energy loss for this zone is ${}_{\Delta}\mathrm{T}_{\Sigma\min} = 210.1979[\mathrm{J}]$. The values of the two parameters at which the loss of kinetic energy assumes minimum values were also obtained, which are e=0.0003[m]. α=0.0052 [rad]. These values can be used in the design of new band saw machines in order to guarantee minimum energy consumption. They can also be used for band saw machines that perform technological operations. In this case, it is necessary to measure the actual values of the two parameters and compare them with the optimal values calculated using the optimization procedures. In this way, we can determine whether the band saw machine is working in optimal mode or not. If the energy loss is very large, technological solutions must be applied to reduce the sizes of e and α.

The initial zone is the most desirable zone in which the band saw machine can operate. Theoretically, the energy loss in this zone can be zero. This can be obtained when there are no linear and angular deviations, i.e. $e = 0, \quad \alpha = 0, \quad {}_{\Delta}\mathrm{T}_{\Sigma\min} = 0$. Unfortunately, this is technologically impossible. The actual value calculated by the optimization procedures is ${}_{\Delta}\mathrm{T}_{\Sigma\min} = 3.0272\ [\mathrm{J}]$ when α=0, $e = 1.0e - 003 * 0.3000\ [m]$. The parameter values are also hard to achieve, which means that the band saw machine is very difficult to work in this zone.

The most undesirable zone in which the band saw machine can operate is the final zone. In this zone, the loss of kinetic energy assumes maximum value, which is ${}_{\Delta}\mathrm{T}_{\Sigma\min} = 5.2504\mathrm{e} + 003[\mathrm{J}]$. This value is calculated for the maximum values of the two parameters, $e = 0.0015\ [m], \quad \alpha = 0.026175\ [rad]$. The actual value calculated from the optimization procedure is ${}_{\Delta}\mathrm{T}_{\Sigma\min} = 2.7856\mathrm{e} + 003[\mathrm{J}]$, when $e = 1.0e - 003 * 0.3000\ [m]$ and $\alpha = 0.0209\ [rad]$. If the band saw machine works in this area, technological solutions should be applied, the sizes of linear and angular inaccuracies should be reduced and the machine should work in the working zone.

3. Conclusion

In this paper, the loss of kinetic energy in big band saw machines are examined. The main objectives of the study are formulated and the corresponding main tasks are solved. Expressions have been obtained to calculate the kinetic energy of the mechanical systems in the ideal and the real case. With the help of these expressions, the final dependencies for determining the energy losses for the studied class of machines were obtained. These dependences show the influence of linear and angular inaccuracies, i.e. of the parameters e and α. A number of optimization solutions have been proposed that allow calculating the values of both parameters that ensure a minimum value of energy loss.

In conclusion, this research can be seen as a contribution to the contemporary knowledge on energy conservation and design of energy-saving machines. It may contribute to better understanding of these problems and prove useful for many researchers. In particular, the proposed approach can be used in the design of other classes of big woodworking machines, as well as in the study of energy losses for this class of machines.

Compliance with ethical standards

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Disclosure of conflict of interest

The author of this article declares that he has no conflict of interest with anyone in conducting and completing this study.

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