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NONLINEAR DYNAMIC MODELING AND PID ATTITUDE CONTROL OF A QUAD (+) CONFIGURATION UAV: A MATLAB-BASED SIMULATION STUDY

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ABSTRACT

Quadrotor unmanned aerial vehicles (UAVs) are increasingly deployed in aerial photography, search and rescue, surveillance, and logistics applications, yet their inherently unstable and nonlinear dynamics make controller design and validation a central challenge. This paper presents the derivation of a full twelve-state nonlinear dynamic model for a Quad(+) configuration UAV and the design of a cascaded proportional-integral-derivative (PID) attitude controller for roll, pitch, yaw, and vertical-velocity regulation. The model incorporates rigid-body translational and rotational equations of motion expressed in Euler-angle parameterization, together with the thrust and moment allocation relating individual rotor speeds to total thrust and body-axis moments. The controller and plant were implemented and simulated in MATLAB over a two-second horizon subject to step commands of −10° roll, 10° pitch, 10° yaw, and 1 m/s descent rate. Simulation results show that all four attitude channels converge to their commanded values, with the roll response tracking without overshoot and the pitch, yaw, and vertical-velocity channels exhibiting bounded overshoot of approximately 10–20% before settling within 1.2–1.5 seconds. These results confirm that a linear PID scheme, despite neglecting the higher-order nonlinearities of the plant, can provide adequate attitude regulation near hover for a Quad(+) UAV, while also revealing the coupling between attitude and lateral translation that motivates more advanced nonlinear control strategies for aggressive maneuvers.

Keywords: Quadrotor; UAV; PID control; Attitude control; Nonlinear dynamics; MATLAB simulation; Euler angles

1 INTRODUCTION

The quadrotor (QR) is a type of unmanned aerial vehicle that has become increasingly popular in recent years, with active research into applications including search and rescue, surveillance, reliable delivery of food and medicine in emergency situations, and object manipulation in construction and transportation. Quadrotors have already been demonstrated to be useful for multi-agent missions, mapping, exploration, transportation, and entertainment purposes such as acrobatic aerial performances.

Designing and controlling a quadrotor is a complex task that requires a thorough understanding of its dynamics and of the control algorithms used to stabilize it. Typically, linear controllers are used, with the system dynamics described by a set of linear differential equations obtained by linearizing the nonlinear plant around an equilibrium point (commonly hover). This linear approach is widely adopted because it circumvents much of the complexity involved in modeling and controlling the true nonlinear behavior of the vehicle; however, it restricts operation to small-angle maneuvers and is not well suited to applications that demand quick, aggressive flight.

Nonlinear control techniques, by contrast, can more realistically capture quadrotor dynamics and are often based on nonlinear differential equations expressed directly on the vehicle's configuration manifold. Nonlinear geometric control, for example, formulates the dynamics in a coordinate-free manner, avoiding singularities such as gimbal lock that arise in Euler-angle representations and enabling controllers with almost-global convergence guarantees. Nonetheless, because of their simplicity, ease of tuning, and adequate performance near hover, PID controllers remain the most widely used attitude-control scheme in both commercial and research quadrotor platforms, and a rigorous simulation-based characterization of their performance is a necessary first step before more advanced schemes are pursued.

This paper addresses that need. We derive the full twelve-state nonlinear dynamic model of a Quad(+) configuration UAV, describe the design of a cascaded PID attitude controller operating on the linearized near-hover dynamics, and evaluate the closed-loop performance through MATLAB simulation of representative roll, pitch, yaw, and vertical-velocity step commands. The remainder of the paper is organized as follows. Section 2 presents the vehicle configuration, nomenclature, and nonlinear equations of motion. Section 3 describes the design of the PID attitude controller. Section 4 details the simulation setup. Section 5 presents and discusses the simulation results. Section 6 concludes the paper and outlines directions for future work.

2 VEHICLE CONFIGURATION AND DYNAMIC MODEL

2.1 Quad(+) Airframe Configuration

The vehicle studied is a symmetric four-rotor UAV in the "plus" (+) configuration, in which the four motors are distributed along the front, left, back, and right arms of the airframe, each separated by 90°. Table 1 summarizes the motor layout and rotation directions; motors 1 and 3 (front and back) rotate counterclockwise while motors 2 and 4 (left and right) rotate clockwise, so that the reaction torques of adjacent rotors cancel in steady hover.

Table 1: Quad(+) motor placement and rotation directions.

Motor #	Location	Rotation direction	Notes
1	Front	Counter-clockwise	$O_1 = +1$, $\xi_1 = 0°$
2	Left	Clockwise	$O_2 = -1$, $\xi_2 = 270°$
3	Back	Counter-clockwise	$O_3 = +1$, $\xi_3 = 180°$
4	Right	Clockwise	$O_4 = -1$, $\xi_4 = 90°$

The nominal plant specifications used throughout the simulation are listed in Table 2, together with the parameter uncertainties that may be applied to test controller robustness.

Table 2: Nominal plant specifications

Specification	Value	Uncertainty
Mass, m	1.25 kg	±10%
Arm length, l	0.265 m	±0.01
x moment of inertia, Ixx	0.0232 kg·m^2	±0.01
y moment of inertia, Iyy	0.0232 kg·m^2	±0.01
z moment of inertia, Izz	0.0468 kg·m^2	±0.01

2.2 State Variables and Reference Frames

The vehicle state is described in an inertial North-East-Down (NED) frame {A} and a body-fixed Front-Right-Down frame {B}. Twelve state variables are used, comprising position, linear velocity, Euler attitude angles, and body-axis angular rates:

$$\vec{x} = [X, Y, Z, dX/dt, dY/dt, dZ/dt, \varphi, \theta, \psi, p, q, r]^T$$

where (X, Y, Z) is the position of the airframe in {A}, (φ, θ, ψ) are the roll, pitch, and yaw Euler angles, and (p, q, r) are the body-axis angular rates. All twelve states are assumed observable, consistent with the information typically available from a modern GPS/INS sensor suite; sensor noise and the effects of raw-measurement filtering (e.g., via a Kalman filter) are not modeled in this study.

Four control inputs are used: the total thrust and the three body-axis moments,

$$\vec{u} = [u_1, u_2, u_3, u_4]^T = [T\Sigma, M_1, M_2, M_3]^T$$

2.3 Thrust and Moment Allocation

Each rotor produces a thrust Ti proportional to the square of its angular speed Ωi, $Ti = kT\Omega i^2$, where kT is a lumped thrust coefficient. For the symmetric Quad(+) airframe, the total thrust and the three body-axis moments are related to the four rotor speeds by

$$[T\Sigma, M_1, M_2, M_3]^T = A\cdot[\Omega_1^{\,2}, \Omega_2^{\,2}, \Omega_3^{\,2}, \Omega_4^{\,2}]^T$$

where A is the 4×4 allocation matrix built from kT, kQ (the reaction-torque coefficient), and the arm length l. Inverting this relation gives the rotor speeds required to produce a desired thrust and moment set, which is the standard mixing step between the attitude controller output and the individual motor commands.

2.4 Nonlinear Equations of Motion

Applying Newton's second law in the inertial frame gives the translational dynamics

$$d\vec{r}/dt = \vec{v}, \qquad m\cdot d\vec{v}/dt = mg\cdot\vec{a}_3 + R\cdot T\Sigma\cdot(-\vec{z})$$

where R is the rotation matrix from the body frame to the inertial frame and $\vec{a}_3$ is the inertial-frame Down unit vector. Expanding the acceleration components using the 3-2-1 (yaw-pitch-roll) Euler-angle parameterization yields

$$d^2X/dt^2 = -(1/m)(\sin\psi\cdot\sin\varphi + \cos\psi\cdot\sin\theta\cdot\cos\varphi)\cdot T\Sigma$$

$$d^2Y/dt^2 = -(1/m)(-\cos\psi\cdot\sin\varphi + \sin\psi\cdot\sin\theta\cdot\cos\varphi)\cdot T\Sigma$$

$$d^2Z/dt^2 = -(1/m)\cos\theta\cdot\cos\varphi\cdot T\Sigma + g$$

The rotational kinematics relating the Euler-angle rates to the body-axis angular rates are

$$d\varphi/dt = p + \sin(\varphi)\tan(\theta)\cdot q + \cos(\varphi)\tan(\theta)\cdot r$$

$$d\theta/dt = \cos(\varphi)\cdot q - \sin(\varphi)\cdot r$$

$$d\psi/dt = \sin(\varphi)\sec(\theta)\cdot q + \cos(\varphi)\sec(\theta)\cdot r$$

and the rotational dynamics, obtained from the Euler rigid-body equations with diagonal inertia matrix I = diag(Ixx, Iyy, Izz), are

$$dp/dt = [(Iyy - Izz)/Ixx]\cdot qr + M_1/Ixx$$

$$dq/dt = [(Izz - Ixx)/Iyy]\cdot rp + M_2/Iyy$$

$$dr/dt = [(Ixx - Iyy)/Izz]\cdot pq + M_3/Izz$$

The equations for d^2X/dt^2, d^2Y/dt^2, and d^2Z/dt^2 above are visibly nonlinear and coupled in φ, θ, ψ, which is what motivates the use of nonlinear or geometric control formulations for large-angle, aggressive maneuvers; near hover, however, these equations may be linearized to support classical PID design, as described in Section 3.

Additional subsystems — motor dynamics, battery discharge behavior, electronic speed controller response, gyroscopic coupling, and wind disturbances — can be appended to this core rigid-body model. In this study, motor dynamics are represented by a first-order lag with time delay, illustrating how actuator bandwidth can be incorporated into the simulation without altering the rigid-body equations above.

3 PID ATTITUDE CONTROLLER DESIGN

The controller architecture follows a cascaded structure in which an outer trajectory planner and position controller generate the desired thrust and attitude commands (φcmd, θcmd, ψcmd), which are then tracked by an inner attitude controller that computes the moments u_2, u_3, u_4 required to drive the vehicle to the commanded orientation, while a separate vertical-velocity loop regulates u_1. This work focuses on the inner attitude loop and the vertical-velocity loop, assuming the vehicle operates near hover so that a linear PID design is applicable.

Because the positive-Z direction in the local NED frame points downward, a negative sign appears after the mg term in the thrust command to account for the sign convention. The four PID control laws implemented are

$$T\Sigma = mg - [K_{P,\dot{z}}(\dot{z}_{cmd}-\dot{z}) + K_{I,\dot{z}}\int(\dot{z}_{cmd}-\dot{z})dt + K_{D,\dot{z}}(d\dot{z}_{cmd}/dt - d\dot{z}/dt)]$$

$$M_1 = K_{P,\varphi}(\varphi_{cmd}-\varphi) + K_{I,\varphi}\int(\varphi_{cmd}-\varphi)dt + K_{D,\varphi}(p_{cmd}-p)$$

$$M_2 = K_{P,\theta}(\theta_{cmd}-\theta) + K_{I,\theta}\int(\theta_{cmd}-\theta)dt + K_{D,\theta}(q_{cmd}-q)$$

$$M_3 = K_{P,\psi}(\psi_{cmd}-\psi) + K_{I,\psi}\int(\psi_{cmd}-\psi)dt + K_{D,\psi}(r_{cmd}-r)$$

where the derivative terms are computed either from the measured body-axis rates directly (roll channel) or from the finite-difference rate of the tracking error (pitch and yaw channels), and the integral terms accumulate the tracking error at each simulation step. The gains used in this study, obtained through manual tuning to achieve acceptable settling time and overshoot near hover, are listed in Table 3.

Table 3: PID controller gains used in simulation.

Channel	KP	KI	KD
Roll (φ)	0.20	0.0	0.15
Pitch (θ)	0.20	0.0	0.15
Yaw (ψ)	0.80	0.0	0.30
Z-velocity (ż)	10.0	0.2	0.0

The resulting moment and thrust commands are converted to individual rotor-speed commands via the inverse of the allocation matrix introduced in Section 2.3, which are in turn applied to the plant model through the motor-dynamics subsystem.

4 SIMULATION SETUP

The nonlinear plant and PID controller described above were implemented in MATLAB using an object-oriented Drone class encapsulating the vehicle state, parameters, and control logic, integrated forward in time using explicit Euler integration with a fixed step size of $\Delta t = 0.01$ s. The simulation horizon was set to 2 seconds. Starting from a hover condition at (X, Y, Z) = (0, 0, −5) m with zero velocity and zero attitude, the vehicle was commanded to simultaneously track a step reference of −10° roll, +10° pitch, +10° yaw, and +1 m/s descent rate (ż cmd), exercising all four control channels concurrently. The plant parameters and controller gains used are those listed in Tables 2 and 3.

5 RESULTS AND DISCUSSION

Figure 1 shows the closed-loop time response of the roll, pitch, and yaw angles (top row) and the X, Y position and Z-velocity (bottom row) over the two-second simulation horizon.

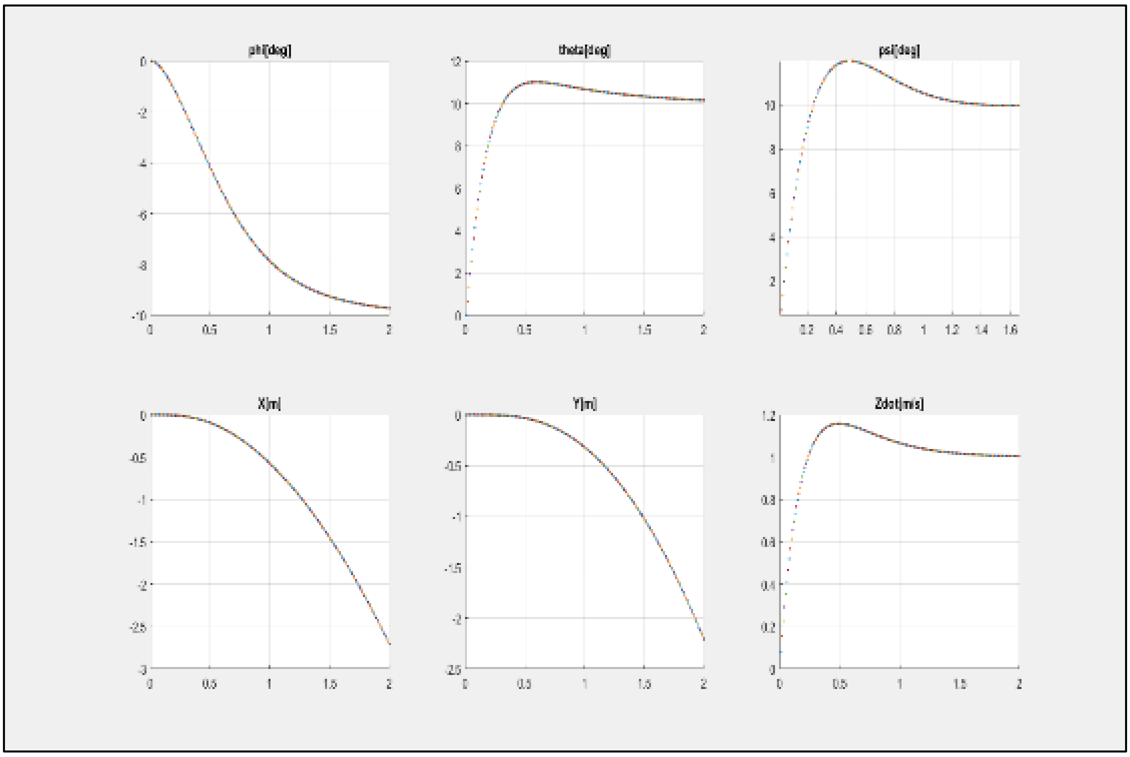

Figure 1: Closed-loop simulation results: roll (φ), pitch (θ), and yaw (ψ) attitude tracking (top row), and X, Y position and vertical velocity (Ż) response (bottom row) under simultaneous step commands of −10° roll, 10° pitch, 10° yaw, and 1 m/s descent rate.

The roll channel (φ) tracks its −10° command with a smooth, non-oscillatory response, reaching approximately −9.7° by t = 2 s without visible overshoot, consistent with the purely proportional-derivative structure used on this channel ($K_{I,\varphi} = 0$). The pitch channel (θ) rises quickly toward its 10° command, overshoots to approximately 11° (≈10% overshoot) near t = 0.5–0.6 s, and settles to approximately 10.2° by t ≈ 1.2 s. The yaw channel (ψ) exhibits the largest transient, overshooting to approximately 12° (≈20% overshoot) near t = 0.4–0.5 s before decaying to approximately 10° by t ≈ 1.5 s — a larger overshoot consistent with its comparatively higher proportional gain ($K_{P,\psi} = 0.8$). The Z-velocity channel similarly overshoots its 1 m/s command, peaking near 1.15 m/s (≈15% overshoot) around t ≈ 0.5 s before settling to approximately 1.0–1.05 m/s, reflecting the combined action of the proportional and integral gains on that channel.

A summary of the approximate transient-response metrics read from Figure 1 is given in Table 4.

Table 4: Approximate transient-response metrics (read from simulation plots).

Channel	Commanded value	Peak / overshoot	Approx. settling time
Roll, φ	−10°	No overshoot	≈ 1.5 s
Pitch, θ	10°	≈11° (≈10%)	≈ 1.2 s
Yaw, ψ	10°	≈12° (≈20%)	≈ 1.5 s
Z-velocity, ż	1.0 m/s	≈1.15 m/s (≈15%)	≈ 1.5 s

Because the attitude commands were applied simultaneously rather than one channel at a time, the X and Y position plots also reveal the coupling inherent in the nonlinear translational dynamics of Section 2.4: the commanded roll and pitch angles tilt the thrust vector, producing a lateral acceleration that drives the vehicle to approximately −2.7 m in X and −2.2 m in Y by t = 2 s, even though no explicit position command was given. This behavior illustrates why, in a complete flight-control system, an outer position-control loop is required to translate desired positions into the small attitude commands that an inner PID attitude loop such as the one presented here can track, and it further motivates the comparison — left for future work — between this linear PID scheme and nonlinear geometric control formulations that remain valid outside the small-angle, near-hover regime assumed in this design.

Overall, the results confirm that a classically tuned PID controller, operating on the linearized near-hover dynamics, is capable of driving all four attitude and vertical-velocity channels of a Quad (+) UAV to their commanded values within approximately 1.2–1.5 seconds and with bounded overshoot not exceeding about 20%, using only proportional and derivative action on two of the four channels and no more than a single integral term elsewhere.

6 CONCLUSION

This paper has presented the derivation of a twelve-state nonlinear dynamic model for a Quad (+) configuration UAV and the design and MATLAB simulation of a cascaded PID attitude controller regulating roll, pitch, yaw, and vertical velocity near hover. Simulation of simultaneous step commands across all four channels showed convergence to the commanded values within 1.2–1.5 seconds, with overshoot ranging from none (roll) to approximately 20% (yaw) and revealed the coupling between commanded attitude and resulting lateral translation that is characteristic of the underlying nonlinear equations of motion. These findings support the continued use of PID control for near-hover attitude regulation while underscoring the need for nonlinear or geometric control approaches, together with an outer position-control loop, for aggressive or large-angle maneuvers. Future work will extend the present model to include motor and battery subsystem dynamics explicitly in the reported results, incorporate sensor-noise and estimation effects via a Kalman filter, and provide a direct quantitative comparison between this PID design and a nonlinear geometric attitude controller under identical simulation conditions.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

The authors declare no financial or non-financial conflicts of interest with respect to the research, authorship, and publication of this article.

REFERENCES

[1] Groves PD. Principles of GNSS, inertial, and multisensor integrated navigation systems. Boston (MA): Artech House; 2013.

[2] Leishman GJ. Principles of helicopter aerodynamics. Cambridge: Cambridge University Press; 2006.

[3] Siravuru A. Geometric-toolbox for MATLAB [software repository]. 2013.

[4] Sreenath K. RI seminar: Nonlinear geometric control [video]. 2013.

[5] Lee T, Leok M, McClamroch NH. Geometric tracking control of a quadrotor UAV on SE(3). In: Proceedings of the 49th IEEE Conference on Decision and Control (CDC); 2010. p. 5420–5425.

[6] Lee T, Leok M, McClamroch NH. Control of complex maneuvers for a quadrotor UAV using geometric methods on SE(3). IEEE; 2011.

[7] Vo N. Modeling and control of a quadrotor UAV [MSc thesis]. Delft: TU Delft, Faculty of Mechanical Engineering; 2017.