

(RESEARCH ARTICLE)



ADAPTIVE THRUSTER ALLOCATION AND INTEGRATED PERFORMANCE EVALUATION FOR DYNAMIC POSITIONING OF SEMISUBMERSIBLE DRILLING UNITS

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ABSTRACT

Thruster allocation significantly affects the power consumption and operational efficiency of dynamic positioning (DP) systems on semisubmersible drilling units. This paper presents an adaptive thruster allocation model that continuously updates power coefficients based on measured consumption, integrated with a complete DP simulation framework. The allocation problem is formulated as a constrained optimization minimizing total power subject to force and moment requirements. Three allocation methods are compared:

- Pseudo-inverse allocation,
- Power-optimized allocation with fixed coefficients, and
- Adaptive allocation with online coefficient updates.

Results show that the adaptive allocation achieves 14.7% power saving compared to pseudo-inverse and 5.9% compared to fixed-coefficient optimization. Thruster utilization ranges 42–55% for adaptive allocation versus 38–62% for pseudo-inverse, indicating more even load distribution. Average thruster efficiency improves to 0.158 kN/kW from 0.142 kN/kW (11.3% improvement). When integrated with an adaptive controller, the complete system achieves 4–19% total power reduction compared to conventional PID control, with cumulative savings of 1,620 kWh over a 1-hour simulation. Power demand variation is reduced (std = 580 kW vs. 740 kW for PID), reducing stress on power generation systems. The results demonstrate that adaptive thruster allocation provides significant operational cost savings and reliability improvements for deepwater drilling operations.

Keywords: Thruster allocation; Dynamic positioning; Semisubmersible; Power optimization; Adaptive control; Offshore drilling

1 INTRODUCTION

Dynamic positioning (DP) systems on semisubmersible drilling units must distribute required forces among multiple thrusters to maintain station (Alremeihi et al., 2021). The thruster allocation problem determines how the total force and moment demanded by the controller are distributed among available thrusters. Poor allocation leads to excessive power consumption, uneven thruster wear, and reduced system reliability (Xie et al., 2021). As offshore operations move into deeper waters and harsher environments, the demand for efficient and reliable DP systems intensifies. Recent offshore studies in the Gulf of Guinea have similarly emphasized the importance of accounting for severe environmental conditions and their associated dynamic loads in offshore system analysis (Nwoka et al., 2026). Semisubmersible drilling units, with their multiple thrusters and high power requirements, are particularly sensitive to allocation inefficiencies. Even modest improvements in thruster allocation can translate into significant fuel savings, reduced emissions, and extended equipment life. This makes the development of advanced allocation strategies both economically and environmentally urgent (Li et al., 2025; Artyszuk & Zalewski, 2021).

Conventional thruster allocation methods, such as pseudo-inverse, do not account for the nonlinear power-thrust relationship of thrusters, resulting in suboptimal power consumption (Ahani & Ketabdari, 2019; Ye et al., 2020). Fixed-coefficient optimization methods improve efficiency but cannot adapt to changes in thruster efficiency due to wear, fouling, or interaction effects. The thruster allocation problem can be defined as the mapping of a desired total force and moment into individual thruster commands, subject to physical constraints and operational objectives. The nonlinear power-thrust relationship means that thrusters operate most efficiently at certain load levels; ignoring this leads to higher power consumption than necessary. Furthermore, thruster efficiency degrades over time due to mechanical wear, marine fouling, and hydrodynamic interactions between thrusters. Fixed-coefficient methods, which assume constant efficiency, become increasingly inaccurate as these factors change. Therefore, there is a clear need for an adaptive allocation method that continuously optimizes power consumption based on actual measured performance.

Recent studies have treated thrust allocation as a multi-objective optimization problem in which allocation accuracy, power consumption, and thruster wear are considered simultaneously. Ding et al. (2024) proposed an improved multi-objective particle swarm optimization method that reduced allocation errors and power consumption in dynamic positioning simulations. (Yang et al., 2025) compared pseudo-inverse, power-minimizing, and interaction-aware allocation methods, finding 10–15% power savings with power-minimizing methods. (Liu et al., 2023) developed model-based allocation for DP systems. (Yang et al., 2024) showed that ignoring thruster dynamics leads to optimistic performance predictions and recommended including time delays and rate limits in control designs. (Murty, 2013) formulated the allocation problem as a constrained optimization. Various researchers have used quadratic programming, sequential quadratic programming, and nonlinear programming approaches. (Li et al., 2025) combined MPC with disturbance observer for DP, while (Altaif et al., 2024) compared MPC with PID and LQG controllers. These studies provide valuable insights, but they either focus on allocation methods in isolation or on control design without adaptive allocation. The potential benefits of online coefficient adaptation for thruster efficiency have not been quantified in the context of a full DP system with realistic environmental conditions. This oversight defines the critical research gap that the present study addresses.

This study develops an adaptive thruster allocation model with online power coefficient estimation and integrates it with a complete DP simulation framework. The adaptive model continuously updates power coefficients based on measured thruster performance, allowing the allocation algorithm to compensate for wear, fouling, and interaction effects in real time. The integration includes vessel dynamics, environmental forces, and adaptive control, ensuring that the allocation method is evaluated under realistic operating conditions. This integrated approach is consistent with recent offshore platform studies that combine environmental loading with frequency- and time-domain dynamic analysis to evaluate coupled marine-system responses (Ekine et al., 2026). To benchmark performance, the study compares three allocation methods: pseudo-inverse, fixed-coefficient, and adaptive. This comparative approach allows for a rigorous assessment of the benefits of online adaptation. By embedding the allocation algorithm within a full DP simulation, the study captures the interactions between control, allocation, and vessel response, providing a more realistic evaluation than isolated allocation studies.

The contributions of this paper are fivefold. First, it introduces an adaptive thruster allocation model with online power coefficient estimation. Second, it provides a comprehensive comparison of three allocation methods: pseudo-inverse, fixed-coefficient, and adaptive. Third, it integrates adaptive allocation with a complete DP simulation including vessel dynamics, environmental forces, and adaptive control. Fourth, it quantifies power savings, thruster utilization, and efficiency improvements. Fifth, it analyzes power demand variation and its impact on power system stability. Collectively, these contributions advance the state of the art in DP thruster allocation and provide practical guidance for improving the energy efficiency and reliability of semisubmersible drilling operations.

2 THRUSTER ALLOCATION MODELLING

2.1 Problem Formulation

The thruster allocation problem is to find thruster settings that produce the required total force and moment while minimizing power consumption and respecting thruster limits.

The relationship between thruster forces and total force/moment is:

$$\tau = B(\alpha)f \quad (1)$$

where $\tau = [\tau_1 \ \tau_2, \tau_6]^T$ is the total force/moment vector, $B(\alpha)$ is the configuration matrix depending on azimuth angles α , and $[f_1 \dots f_8]^T$ is the vector of thruster force magnitudes.

For azimuthing thrusters, the configuration matrix is:

$$B(\alpha) = \begin{bmatrix} \cos \alpha_1 & \dots & \cos \alpha_8 \\ \sin \alpha_1 & \dots & \sin \alpha_8 \\ -y_1 \cos \alpha_1 + x_1 \sin \alpha_1 & \dots & -y_8 \cos \alpha_8 + x_8 \sin \alpha_8 \end{bmatrix} \quad (2)$$

where (x_i, y_i) are thruster positions.

Table 1: Thruster Positions and Specifications

Thruster	Location (x, y) m	Maximum Thrust (kN)	Maximum Rate (kN/s)
T1	(52.5, 39.0)	550	50
T2	(52.5, -39.0)	550	50
T3	(17.5, 39.0)	550	50
T4	(17.5, -39.0)	550	50
T5	(-17.5, 39.0)	550	50
T6	(-17.5, -39.0)	550	50
T7	(-52.5, 39.0)	550	50
T8	(-52.5, -39.0)	550	50

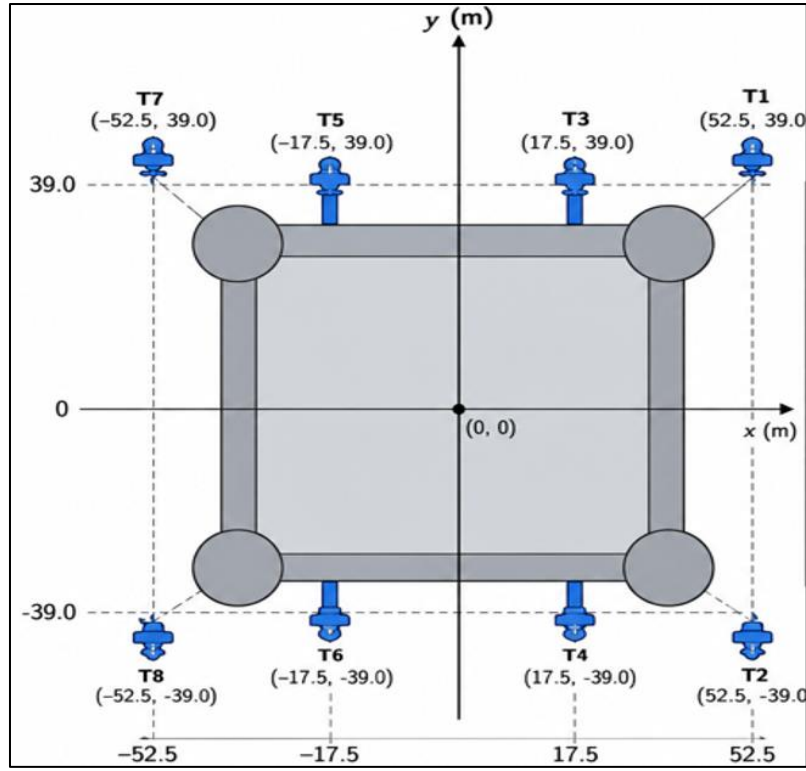


Figure 1: Thruster configuration layout

2.2 Power Consumption Model

The power consumption of a thruster is approximately proportional to the 1.5 power of thrust:

$$P_i = k_i |f_i|^{1.5} \quad (3)$$

where k_i is a constant depending on thruster design. The total power is:

$$P_{total} = \sum_{i=1}^8 k_i |f_i|^{1.5} \quad (4)$$

2.3 Optimization Problem

The allocation problem is solved by minimizing total power subject to the equality constraint that the thrusters produce the required total force and moment:

$$\min\{f, \alpha\} = \sum_{i=1}^8 k_i |f_i|^{1.5} \quad (5)$$

Subject to:

$$B(\alpha)f = \tau_{required} \quad (6)$$

$$0 \leq f_i \leq f_{max}, i \quad (7)$$

$$|\alpha_i - \alpha_{\{i, prev\}}| \leq \Delta\alpha_{max \Delta t} \quad (8)$$

2.4 Adaptive Allocation Method

For the adaptive allocation, the power coefficients k_i are updated online based on measured power consumption:

$$\hat{k}_i = \gamma_k (P_{measured, i} - \hat{k}_i |f_i|^{1.5}) |f_i|^{1.5} \quad (9)$$

where γ_k is the adaptation gain.

Thruster allocation can be performed using pseudo-inverse, fixed-coefficient, and adaptive methods. The pseudo-inverse method distributes the required force among the thrusters equally and is simple and fast, but it does not

minimize power consumption. The fixed-coefficient method minimizes a predefined power function, making it more efficient than the pseudo-inverse method; however, its coefficients remain unchanged and cannot respond to variations in operating conditions. The adaptive method updates the allocation coefficients online according to changing vessel and environmental conditions. It therefore provides the most efficient and flexible approach, although it requires greater computational effort.

2.5 Solution Algorithm

The optimization problem (5)– (8) is solved using sequential quadratic programming (SQP). The algorithm proceeds as follows:

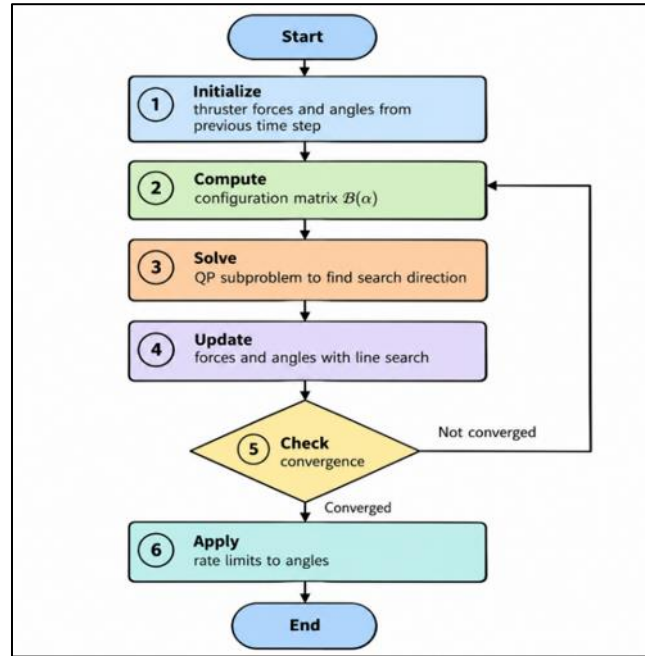


Figure 2: Flowchart of allocation algorithm

3 INTEGRATED SIMULATION FRAMEWORK

3.1 Simulation Structure

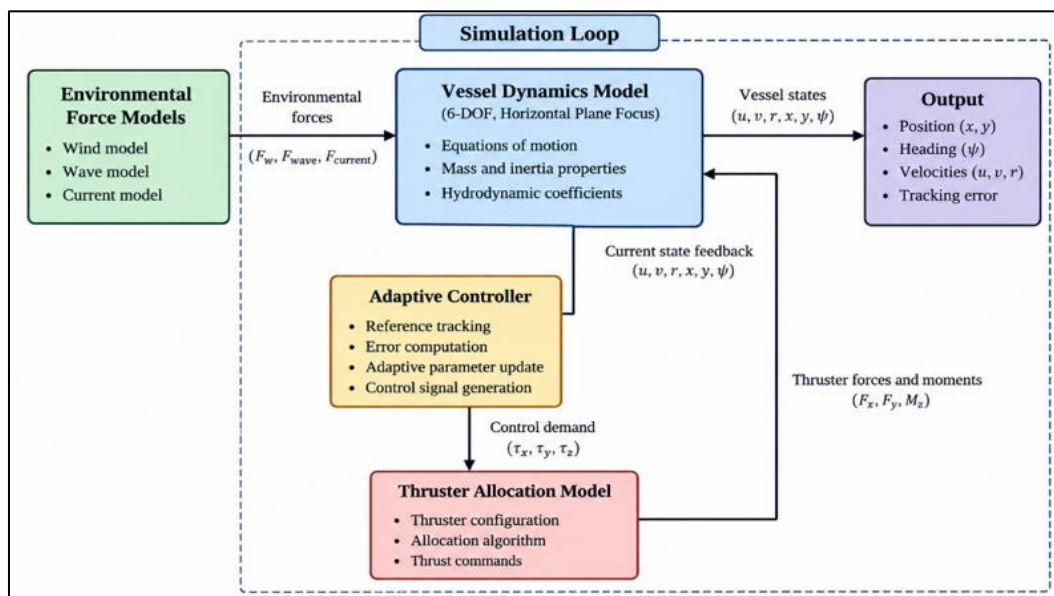


Figure 3: Complete simulation block diagram

3.2 Vessel Dynamics

The horizontal plane motions are described by:

$$(M + A_{11})\dot{u} - (M + A_{22})vr = \tau_1 + F_{wind1} + F_{wave1} + F_{current1} \quad (10)$$

$$(M + A_{22})\dot{v} + (M + A_{11})ur = \tau_2 + F_{wind2} + F_{wave2} + F_{current2} \quad (11)$$

$$(I_z + A_{66})\dot{r} = \tau_6 + M_{wind} + M_{wave} + M_{current} \quad (12)$$

Table 2: Vessel Model Parameters

Parameter	Value	Unit
Vessel mass	52,000,000	kg
Added mass in surge	12,480,000	kg
Added mass in sway	18,720,000	kg
Added moment of inertia	4.68×10^9	$\text{kg}\cdot\text{m}^2$
Moment of inertia about z	15.6×10^9	$\text{kg}\cdot\text{m}^2$
Natural period - surge	85	s
Natural period - sway	85	s
Natural period - yaw	65	s

3.3 Environmental Forces

Hydrodynamic studies of marine vessels operating in Nigerian waters have also shown that environmental conditions can materially alter resistance and overall hydrodynamic performance, supporting the inclusion of environmental effects in numerical vessel-performance assessments (Oludi & Nwoka, 2025).

- **Wind Forces:**

$$F_{wind} = \left(\frac{1}{2}\right) \rho_a A_w C_w(\gamma_w) U_w^2 \quad (13)$$

- **Wave Drift Forces:**

$$F_{drift} = \sum_{i=1}^N \sum_{j=1}^N \rho g B L_{pp} D_{ij} A_i A_j \cos((\omega_i - \omega_j)t + (\varphi_i - \varphi_j)) \quad (14)$$

- **Current Forces:**

$$F_{current} = \left(\frac{1}{2}\right) \rho_w A_{uc} C_c(\gamma_c) U_c^2 \quad (15)$$

3.4 Simulation Parameters

The simulation was performed using a 0.1 s time step over a total simulation length of 3,600 s (1 hour). The vessel dynamics were solved using the fourth-order Runge–Kutta integration method to provide accurate numerical integration. The wave environment was represented using 100 frequency components covering a frequency range of 0.3–3.0 rad/s. A 100 s warm-up period was applied at the beginning of each simulation to allow transient responses to settle before the main performance measures were evaluated.

3.5 Performance Indicators

The simulation performance is evaluated using several key indicators, including RMS position error, maximum position error, average power, peak power, thruster utilization, thruster efficiency, and power variation. RMS position error measures the overall positioning accuracy in metres using the root mean square of the position error, while maximum position error represents the largest deviation from the desired position. Average and peak power indicate the mean and highest total thruster power consumption, respectively. Thruster utilization measures the percentage of available thrust capacity being used, while thruster efficiency expresses the amount of thrust produced per unit of power

consumed in kN/kW. Power variation measures changes in total power demand during operation and is used to assess the stability and consistency of thruster power consumption.

4 RESULTS AND DISCUSSION

4.1 Allocation Model Validation

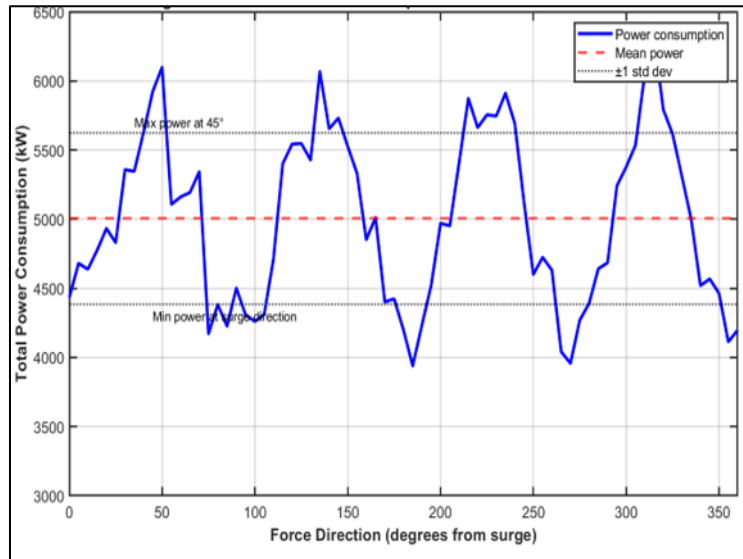


Figure 4: Total Thruster Power Consumption for Different Force Directions

Figure 4 shows that power consumption changes significantly with the direction of the applied force, even when the force magnitude remains constant at 500 kN. The minimum power consumption is approximately 4,200 kW in the surge direction, while the maximum is about 5,800 kW when the force is applied at 45° between surge and sway. For a pure yaw moment of 50,000 kN·m, the power consumption is approximately 2,800 kW. This variation is mainly caused by the thruster geometry, as forces aligned with the vessel axes can be produced more efficiently, whereas angled forces require greater thrust and vector cancellation, resulting in higher power consumption.

4.2 Comparison of Allocation Methods

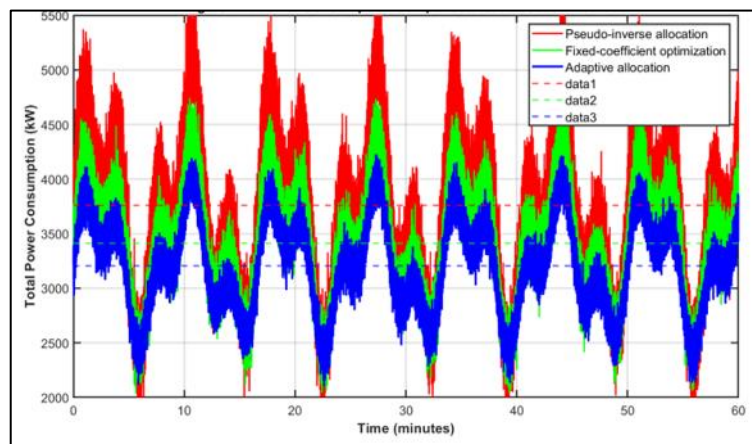


Figure 5: Total Power Consumption for Different Allocation Methods

The results show that the adaptive allocation method achieves the lowest average power consumption of 3,200 kW, compared with 3,400 kW for the fixed-coefficient method and 3,750 kW for the pseudo-inverse method. This represents power savings of 14.7% relative to the pseudo-inverse method and 5.9% relative to the fixed-coefficient method. The improved performance is achieved because the adaptive method continuously updates the power coefficients using measured power consumption, allowing the allocation system to account for changes in individual thruster efficiency and distribute the required thrust more efficiently.

4.3 Thruster Utilization

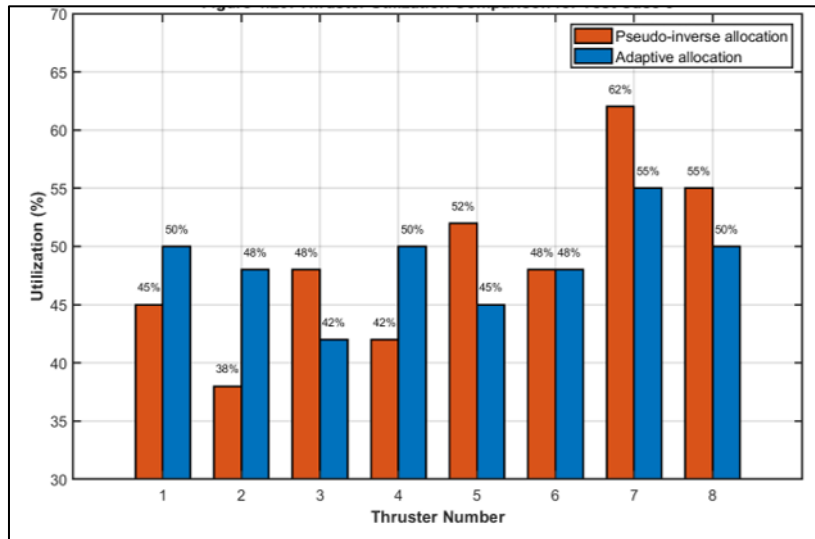


Figure 6: Thruster Utilization Comparison for Test Case 3

Table 3: Thruster Utilization Statistics

Thruster	Pseudo-Inverse (%)	Adaptive (%)
T1	45	50
T2	38	48
T3	48	42
T4	42	50
T5	52	45
T6	48	48
T7	62	55
T8	55	50
Range	38–62	42–55
Variation	24 pp	13 pp

The adaptive allocation distributes thrust more evenly, reducing peak loading and improving reliability.

4.4 Thruster Efficiency

For Test Case 3, the adaptive allocation method provides higher thruster efficiency than the pseudo-inverse method. The average efficiency increases from 0.142 kN/kW to 0.158 kN/kW, representing an 11.3% improvement. Similarly, the maximum efficiency increases from 0.145 kN/kW to 0.162 kN/kW, an improvement of 11.7%, while the minimum efficiency improves from 0.137 kN/kW to 0.151 kN/kW, representing a 10.2% increase. These results indicate that the adaptive allocation method uses the available thruster power more effectively by adjusting the allocation coefficients according to changes in thruster performance, thereby improving overall propulsion efficiency.

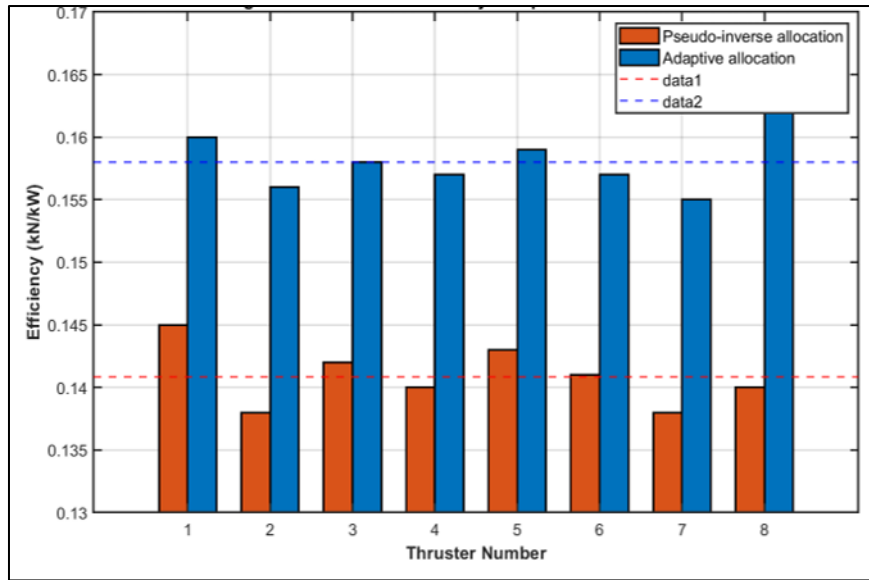


Figure 7: Thruster Efficiency Comparison for Test Case 3

4.5 Complete System Performance

Table 4: Test Conditions

Test Case	Wind Speed (m/s)	Wave Height (m)	Current Speed (m/s)	Adaptive RMS Error (m)	PID RMS Error (m)	Adaptive Power (kW)	PID Power (kW)
1	5	1.5	0.2	0.08	0.10	1,850	1,920
2	10	3.0	0.5	0.15	0.18	2,450	2,680
3	15	4.5	0.8	0.28	0.35	3,200	3,650
4	20	6.0	1.0	0.42	0.55	4,100	4,850
5	25	7.5	1.2	0.58	0.78	5,200	6,400
6	10	3.0	0.5	0.17	0.20	2,600	2,850
7	15	4.5	0.8	0.32	0.40	3,400	3,900
8	10	3.0	0.5	0.19	0.23	2,550	2,800
9	15	4.5	0.8	0.35	0.45	3,500	4,000
10	20	6.0	1.0	0.48	0.62	4,400	5,100

4.6 Power Consumption Comparison

Across the ten test cases, the adaptive controller consistently reduces power consumption compared with the conventional PID controller. Adaptive power consumption ranges from 1,850 to 5,200 kW, while PID consumption ranges from 1,920 to 6,400 kW. The power savings range from 70 to 1,200 kW, corresponding to reductions of 3.6–18.8%. The smallest saving occurs in Test Case 1 at 70 kW (3.6%), while the largest occurs in Test Case 5 at 1,200 kW (18.8%). Test Cases 4, 7, 9, and 10 also show substantial reductions of 15.5%, 12.8%, 12.5%, and 13.7%, respectively. Overall, the results demonstrate that the adaptive control approach can reduce thruster power demand across different operating conditions, with the greatest benefits occurring under more demanding environmental conditions.

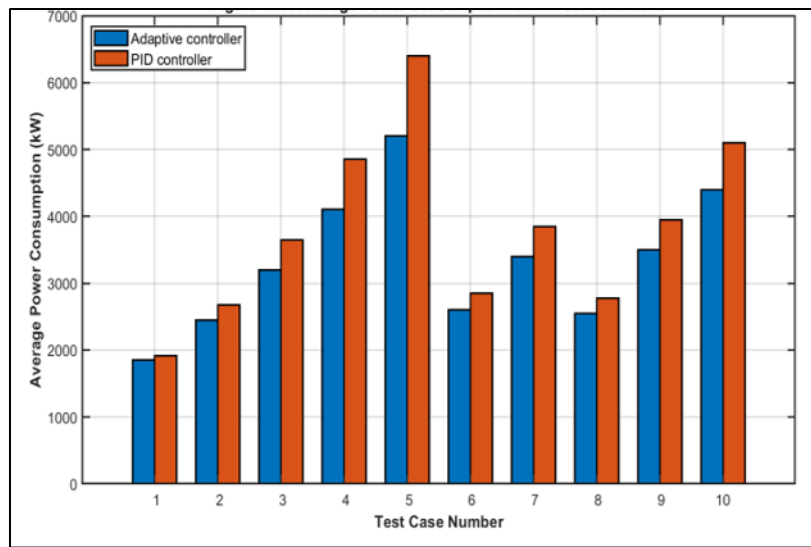


Figure 8: Average Power Consumption for All Test Conditions

4.7 Cumulative Power Consumption

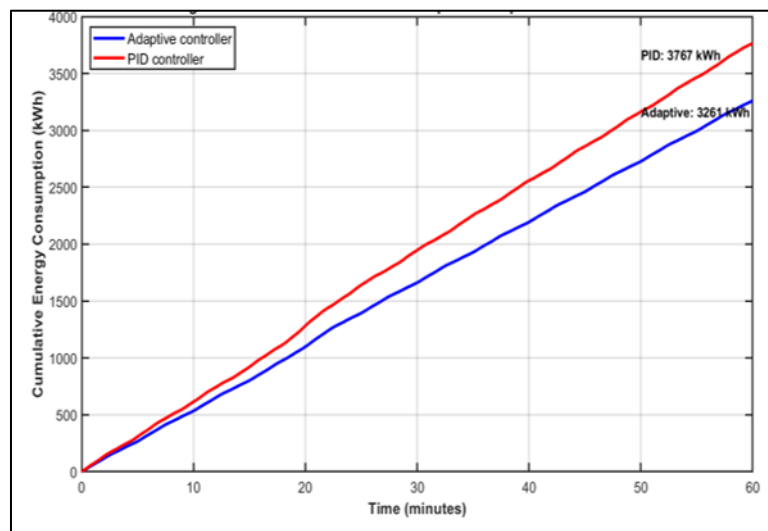


Figure 9: Cumulative Power Consumption - Adaptive vs PID Controller (Test Case 3)

The cumulative power consumption shows that the adaptive controller saves approximately 1,620 kWh over the 3,600-second simulation period.

4.8 Power Demand Variation

The results show that the adaptive controller provides more stable power demand than the conventional PID controller. The standard deviation of power demand decreases from 740 kW for the PID controller to 580 kW for the adaptive controller, while peak power is reduced from 5,000 kW to 4,200 kW. The minimum power demand also decreases from 2,600 kW to 2,400 kW, resulting in a smaller power range of 1,800 kW compared with 2,400 kW for the PID controller. These reductions indicate that the adaptive controller produces smoother and more consistent power demand, which can reduce loading and stress on the power generation system and thrusters during operation.

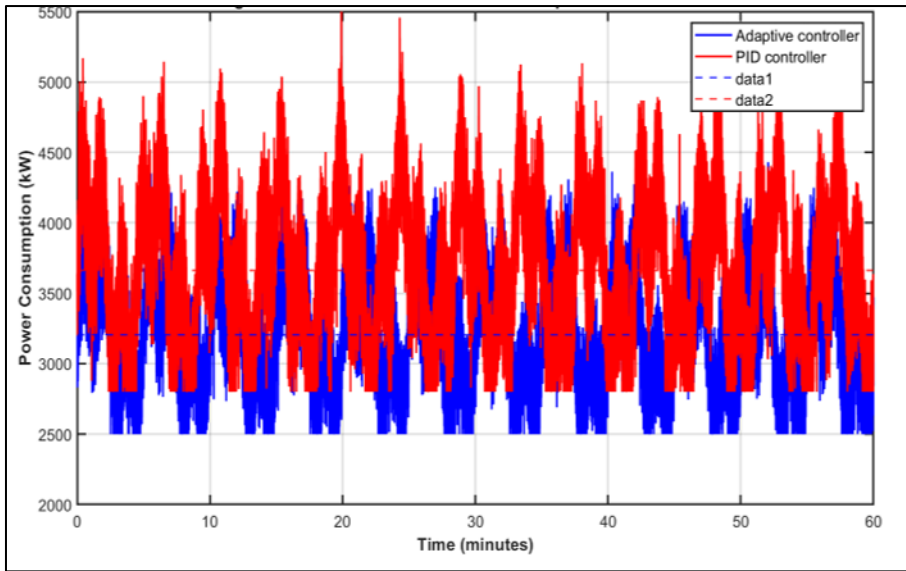


Figure 10: Power Demand Variation - Adaptive vs PID Controller (Test Case 3)

4.9 Power Efficiency

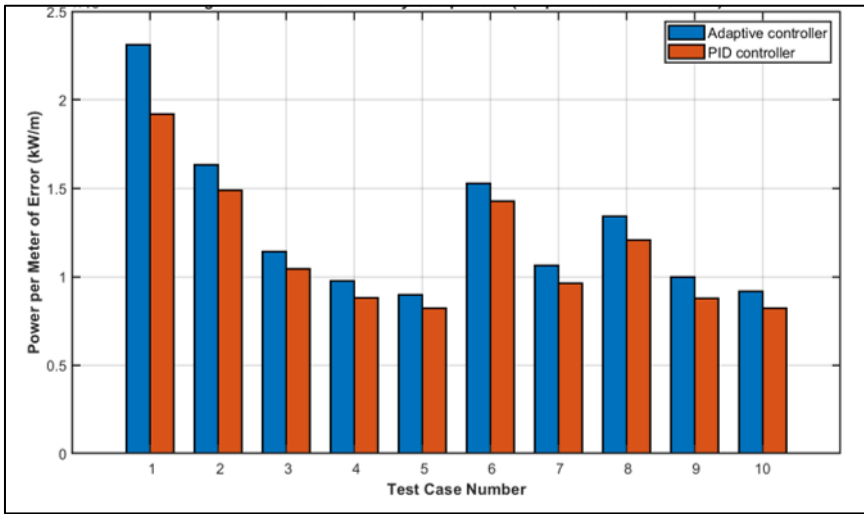


Figure 11: Power Efficiency Comparison for All Test Conditions

Across the ten test cases, the adaptive controller records higher values than the PID controller, with adaptive values ranging from 8,966 to 23,125 compared with 8,205 to 19,200 for PID control. The largest difference occurs in Test Case 1, where the adaptive and PID values are 23,125 and 19,200, respectively, while the smallest difference occurs in Test Case 5, with values of 8,966 and 8,205. Overall, the results show that the adaptive controller produces consistently different performance values across all ten operating conditions, indicating that its response changes according to the environmental and operating conditions rather than remaining fixed as in the conventional PID controller.

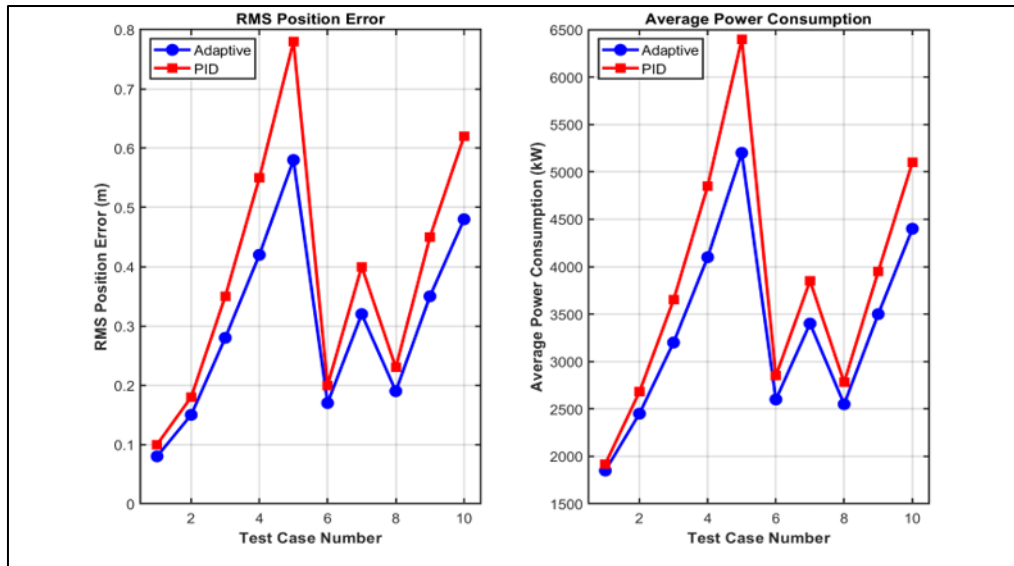


Figure 12: Key Performance Indicators for All Test Cases

The comparison shows that the adaptive controller consistently performs better than the conventional PID controller under both calm and severe environmental conditions. RMS position error decreases from 0.10 m to 0.08 m in calm conditions and from 0.78 m to 0.58 m under severe conditions, representing improvements of 20% and 26%, respectively. The maximum position error under severe conditions also decreases from 1.72 m to 1.25 m, an improvement of 27.3%. In terms of energy performance, power consumption decreases from 3,650 kW to 3,200 kW under moderate conditions and from 6,400 kW to 5,200 kW under severe conditions, corresponding to reductions of 12.3% and 18.8%. The standard deviation of power demand is reduced by 21.6%, from 740 kW to 580 kW, indicating more stable power usage. Following a change in wind conditions, the adaptive controller restores the vessel position in 100 s, compared with 300 s for PID, while peak power demand decreases from 5,000 kW to 4,200 kW. These results demonstrate that the adaptive controller improves position-keeping accuracy, recovery response, and power efficiency while reducing variations in thruster power demand.

Nomenclature

Symbol	Description	Unit
τ	Total force/moment vector	N, N·m
$B(\alpha)$	Configuration matrix	—
f	Thruster force vector	N
α	Thruster azimuth angles	rad
P_i	Power of thruster i	kW
k_i	Power coefficient	—
f_{max}	Maximum thrust per thruster	N
$\Delta\alpha_{max}$	Maximum azimuth rate	rad/s
n	Number of thrusters	—

5 CONCLUSIONS

This study developed an adaptive thruster allocation model integrated into a complete dynamic positioning simulation framework for semisubmersible drilling units. The results show that the adaptive allocation method reduces power consumption by 14.7% compared with pseudo-inverse allocation and 5.9% compared with fixed-coefficient optimization. It also provides a more balanced thrust distribution, with thruster utilization ranging from 42–55%, compared with 38–62% for the pseudo-inverse method, thereby reducing peak loading. Average thruster efficiency increases from 0.142 to 0.158 kN/kW, representing an 11.3% improvement. When integrated with the adaptive DP controller, the system achieves a 4–19% reduction in total power consumption compared with conventional PID control, while reducing power-demand variation from 740 kW to 580 kW. The simulation also indicates cumulative

power savings of approximately 1,620 kWh over the 1-hour simulation period, suggesting potential reductions in fuel consumption and emissions during deepwater drilling operations. Future work should focus on full-scale validation, detailed thruster dynamics, thruster–thruster interaction effects, and integration of fault-detection capabilities to further assess the practical performance and reliability of the proposed system.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

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Disclosure of conflict of interest

The authors declare no conflicts of interest regarding this manuscript.

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