

(RESEARCH ARTICLE)



INVESTIGATION OF ATMOSPHERIC DENSITY VARIATION ON MODEL ROCKET PERFORMANCE

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Global Journal of Engineering and Technology Advances, 2026, 29(01), 244–258

Publication history: Received on 07 September 2026; revised on 17 October 2026; accepted on 19 October 2026

Article DOI: <https://doi.org/10.30574/gjeta.2026.29.1.0298>

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ABSTRACT

Atmospheric density is the environmental parameter to which small-scale rocket altitude performance is most sensitive, yet it is routinely treated as a fixed constant in amateur and educational practice. This paper presents a combined analytical, numerical and literature-based investigation of how variation in air density affects model and high-power rocket flight. A closed-form expression for the coast-phase apogee sensitivity to density is derived and shown to be bounded between zero, in the ballistic limit, and negative unity, in the drag-limited limit, with the governing parameter being the ratio of burnout velocity to terminal velocity. The four physical drivers of density variation — temperature, barometric pressure, water-vapor content and traversed altitude — are quantified separately using the CIPM moist-air formulation, which shows that humidity alone accounts for up to 2.8 % of density at 40 °C and is of the same order as errors practitioners routinely attempt to correct. A three-degree-of-freedom trajectory model with Mach- and Reynolds-dependent drag is integrated for a representative 54 mm high-power vehicle across three measured atmospheric states. A 15.3 % reduction in launch-site density, corresponding to a density altitude of 1692 m, increases predicted apogee by 7.1 %, peak velocity by 1.0 % and coast-phase duration by 4.9 %. The analysis further establishes an asymmetry that is widely misunderstood: static margin is density-invariant, because density cancels from the centre-of-pressure computation, whereas the pitch natural frequency and aerodynamic damping both scale with density and are therefore degraded in low-density conditions. Recommendations for offset-ISA anchoring and Monte Carlo uncertainty propagation in predictive rocketry are given.

Keywords: Aerodynamic Drag, Apogee Prediction, Atmospheric Density, Density Altitude, Dynamic Stability, International Standard Atmosphere, Model Rocketry, Moist Air, Trajectory Simulation.

1 INTRODUCTION

Model and high-power rocketry occupies a flight regime — typically 0 to 3 km above ground level, Mach 0.1 to 1.2, and Reynolds numbers of 10^5 to 10^7 in which aerodynamic drag is not a small perturbation but frequently the dominant force opposing flight [1], [2]. For a representative single-stage vehicle, the integrated drag impulse over a flight can exceed one third of the total motor impulse. Because the drag force scales linearly with air density, and because density at a single launch site can vary by more than fifteen per cent between a cold winter morning at sea level and a hot, humid

afternoon at elevation, atmospheric density is arguably the single most influential environmental parameter governing altitude performance.

Despite this, the prevailing practice in educational and competitive rocketry is to treat density as a constant, most often the International Standard Atmosphere (ISA) sea-level value of 1.225 kg m^{-3} [11], [12]. When a flight fails to reach its predicted altitude, the discrepancy is commonly attributed to motor lot variation, construction finish, or drag coefficient uncertainty. The present work argues that a substantial and predictable fraction of that discrepancy is attributable to the atmosphere itself, and that correcting for it requires no more instrumentation than a thermometer, a barometer and a hygrometer.

The objectives of this paper are fourfold: (i) to establish the analytical relationship between density, drag and rocket kinematics, including a closed-form sensitivity derivation; (ii) to quantify separately the contributions of temperature, pressure, humidity and traversed altitude to density variation; (iii) to determine, by numerical integration of a validated trajectory model, the resulting changes in apogee, peak velocity, coast duration and pitch dynamics; and (iv) to review the standard atmospheric and simulation models available for predictive rocketry and to recommend a density-aware workflow.

2 LITERATURE REVIEW

2.1 Foundational Work

The analytical treatment of drag-inclusive model rocket trajectories originates with Malewicky's altitude prediction charts, issued as Estes Technical Report TR-10 [1]. Malewicky reduced the coast-phase problem to a non-dimensional form governed by the ratio of burnout velocity to terminal velocity, and tabulated the result as a family of charts. Crucially, the charts were computed for a fixed sea-level density, and the report explicitly flags this as a limitation, recommending correction for non-standard conditions without providing the correction method.

Gregorek's companion report TR-11 [2] supplied the drag build-up methodology body skin friction, base drag, fin drag and interference for the same vehicle class, drawing on Hoerner's compendium [14]. Barrowman's NARAM-8 paper [3] and subsequent thesis [4] completed the classical toolkit by providing a slender-body method for locating the centre of pressure. Together, references [1]–[4] constitute the analytical basis on which virtually all subsequent amateur rocketry practice rests.

2.2 Contemporary Simulation Literature

Niskanen's thesis documenting the development of OpenRocket [5] is the most complete modern treatment. It extends the Barrowman method to non-slender and finned configurations, implements a component-wise drag model with Reynolds and Mach dependence, and significant for the present work incorporates an ISA-based atmosphere module with user-settable surface temperature and pressure. Validation against flight data is reported, but the atmospheric sensitivity itself is not the subject of dedicated study.

Box, Bishop and Hunt [6], [7] made the most important methodological contribution for uncertainty-aware prediction. Their six-degree-of-freedom simulator is explicitly stochastic: aerodynamic and environmental parameters are sampled from distributions and propagated to produce a landing-zone probability density rather than a single deterministic trajectory. Their treatment of wind is thorough; atmospheric density, however, is modeled as a deterministic ISA profile.

2.3 Moist-Air Thermophysics

The metrological literature provides the rigorous basis for humid-air density. Davis [19] and the revised CIPM-2007 formulation of Picard et al. [18] give the internationally accepted equation, including compressibility and enhancement factors, with stated uncertainties below 2×10^{-5} in relative terms. Tsilingiris [17] provides validated correlations for the full set of thermophysical and transport properties of humid air between 0 and 100 °C, including the dynamic viscosity required for Reynolds number evaluation. None of this literature has been applied to small-vehicle rocketry in any published study located by the present author.

2.4 Identified Gap

The literature therefore contains a complete analytical trajectory framework [1]–[5], a mature stochastic simulation methodology [6], [7], and a rigorous moist-air density formulation [17]–[19], but no work that combines them. Specifically, no published study quantifies the apogee, velocity, coast-duration and dynamic-stability sensitivity of a model rocket to a fully specified moist-air density state, nor provides a design chart relating density altitude to ejection-delay selection. The present paper addresses the first of these gaps.

3 THEORETICAL FRAMEWORK

3.1 Density and the Equation of State

For dry air treated as a thermally perfect gas, density follows from the ideal gas law [16]:

$$\rho = p / (R_d T) \quad (1)$$

where p is absolute static pressure (Pa), T is absolute temperature (K), and $R_d = 287.058 \text{ J kg}^{-1} \text{ K}^{-1}$ is the specific gas constant for dry air [11]. The vertical structure of the atmosphere follows from hydrostatic equilibrium,

$$dp / dh = - \rho g \quad (2)$$

which, combined with (1) under an assumed constant temperature lapse rate L , yields the standard-atmosphere density profile that underlies the ISA formulation [11], [12]:

$$\rho(h) = \rho_0 (1 - L h / T_0)^{[g / (L R_d) - 1]} \quad (3)$$

with the ISA reference values $T_0 = 288.15 \text{ K}$, $p_0 = 101\,325 \text{ Pa}$, $\rho_0 = 1.225 \text{ kg m}^{-3}$ and $L = 6.5 \text{ K km}^{-1}$ in the troposphere.

3.2 The Drag Force

The aerodynamic drag on the vehicle is conventionally written [8], [15]:

$$D = \frac{1}{2} \rho V^2 C_D A_{ref} \quad (4)$$

where A_{ref} is the reference area, taken as the maximum body cross-section, and C_D is the total drag coefficient, itself the sum of body pressure drag, skin friction, base drag, fin drag and interference contributions [2], [14]. The density therefore enters explicitly through the dynamic pressure,

$$q = \frac{1}{2} \rho V^2 \quad (5)$$

and implicitly through the Reynolds number,

$$Re = \rho V \ell / \mu \quad (6)$$

on which the skin-friction coefficient depends. Since the dynamic viscosity μ is a function of temperature alone, through Sutherland's law, while ρ falls monotonically with altitude, the Reynolds number decreases substantially during ascent and the friction coefficient correspondingly rises. This is a second-order but systematic coupling that a constant- C_D analysis cannot capture [5], [17].

3.3 Kinematics of Powered Flight

The one-dimensional equation of motion for vertical flight is:

$$m(t) dV/dt = T(t) - \frac{1}{2} \rho V^2 C_D A_{ref} - m(t) g \quad (7)$$

with instantaneous mass $m(t) = m_0 - \int \dot{m} dt$ and thrust $T(t)$ taken from a certified motor thrust curve [10]. Because $T(t)$, $m(t)$ and $C_D(M, Re)$ are all time-varying, (7) admits no general closed-form solution and must be integrated numerically [5], [6].

3.4 Closed-Form Coast Phase

The coast phase does admit an analytical solution if ρ , C_D and m are held constant. Defining the drag group

$$k = \frac{1}{2} \rho C_D A_{ref} \quad (8)$$

the altitude gained from burnout velocity V_b to apogee is [1], [27]:

$$h_{coast} = (m / 2k) \cdot \ln [1 + k V_b^2 / (m g)] \quad (9)$$

and the coast duration is:

$$t_{coast} = \sqrt{[m / (g k)]} \cdot \arctan (V_b \sqrt{[k / (m g)]}) \quad (10)$$

A natural velocity scale for the problem is the terminal velocity,

$$V_t = \sqrt{[2 m g / (\rho C_D A_{ref})]} \quad (11)$$

so that the non-dimensional group governing (9) and (10) is $\beta = k V_b^2 / (m g) = (V_b / V_t)^2$. This is precisely Malewicki's chart parameter [1].

3.5 Analytical Density Sensitivity

Differentiating (9) with respect to ρ , noting that $k \propto \rho$ and hence $\beta \propto \rho$, gives the logarithmic apogee sensitivity:

$$\partial \ln h_{coast} / \partial \ln \rho = \beta / [(1 + \beta) \ln(1 + \beta)] - 1 \quad (12)$$

Equation (12) is the central analytical result of this paper and is plotted in Fig. 3. It bounds the behaviour usefully:

As $\beta \rightarrow 0$, corresponding to negligible drag, the sensitivity tends to zero: apogee becomes density-independent, as expected for purely ballistic flight.

As $\beta \rightarrow \infty$, corresponding to heavily drag-limited flight, the sensitivity tends to -1 : apogee varies inversely with density.

Typical model and high-power rockets with V_b / V_t between 1 and 3 fall in the range -0.35 to -0.75 .

Equation (12) accounts only for the coast phase. Because a reduction in density also reduces drag losses during the burn and therefore raises V_b , the full flight sensitivity is larger in magnitude than (12) alone predicts. Section VI quantifies this coupling numerically.

3.6 Dynamic Stability Relations

The classical linearized pitch-oscillation treatment for a passively stabilized rocket [23], [27] gives a corrective moment coefficient

$$C_1 = \frac{1}{2} \rho V^2 A_{ref} C_{N\alpha} (x_{cp} - x_{cg}) \quad (13)$$

and a damping moment coefficient combining aerodynamic and jet-damping terms,

$$C_2 = \frac{1}{2} \rho V A_{ref} \Sigma C_{N\alpha,i} (x_i - x_{cg})^2 + \dot{m} (x_n - x_{cg})^2 \quad (14)$$

from which the pitch natural frequency and damping ratio follow:

$$\omega_n = \sqrt{(C_1 / I_L)}, \quad \zeta = C_2 / [2 \sqrt{(C_1 I_L)}] \quad (15)$$

Two consequences of (13)–(15) are central to Section VI-D. First, $\omega_n \propto V \sqrt{\rho}$, so the pitch response slows as density falls. Second, and in sharp contrast, the centre-of-pressure location x_{cp} obtained from the Barrowman method [3], [4] is a function of geometry and Mach number only: because both the normal force and the pitching moment scale with q , the density term cancels identically from the centre-of-pressure computation. Static margin in calibers is therefore density-invariant while dynamic response is not.

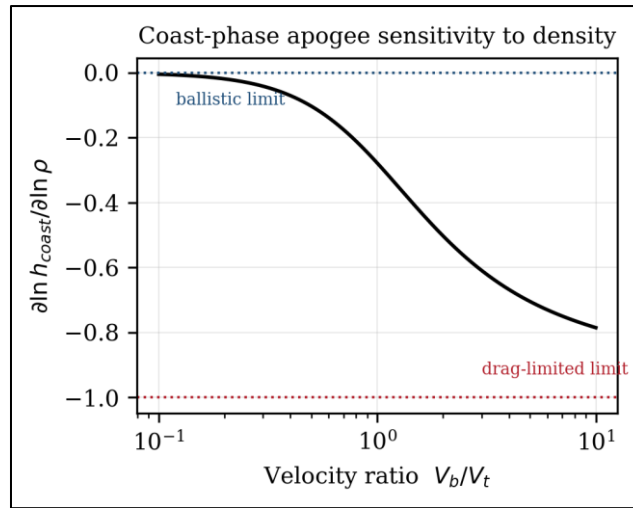


Figure 3: Coast-phase apogee sensitivity to air density, from (12), plotted against the velocity ratio V_b/V_t . The sensitivity is bounded between the ballistic limit of zero and the drag-limited limit of -1 . Typical model and high-power rockets occupy the central portion of the curve.

4 ATMOSPHERIC DENSITY VARIABLES

4.1 Temperature

From (1), $\rho \propto 1/T$, so the fractional sensitivity is exactly -1 :

$$\Delta\rho/\rho = -\Delta T/T \quad (16)$$

A 10 K change about a 293 K baseline therefore produces a 3.4 % density change. Diurnal swings of 15 to 20 K are routine at inland continental and Sahelian launch sites, giving 5 to 7 % density variation at a single site within a single day. Surface superheating over dark, dry launch fields can create a near-surface layer several kelvin warmer than the ambient profile, precisely in the altitude band where the vehicle experiences peak dynamic pressure [20].

4.2 Barometric Pressure

From (1), $\rho \propto p$, and the fractional sensitivity is exactly $+1$. Synoptic pressure variation of ± 20 hPa about 1013 hPa gives ± 2 % density. Site elevation, however, dominates this term: a field at 700 m elevation sits at approximately 932 hPa, some 8 % below sea level, a far larger effect than any weather system. It must be emphasized that station pressure, not the sea-level-corrected QNH reported by aviation and weather services, is the quantity required in (1). Substituting QNH is a common and systematic error that removes the elevation effect entirely.

4.3 Water-Vapor Content

Water vapor has molar mass 18.02 g mol^{-1} against 28.96 g mol^{-1} for dry air. Humid air is therefore less dense than dry air at the same temperature and pressure — a result that is counterintuitive to many practitioners, who associate humidity with “heavy” air. The working form is [19], [28]:

$$\rho = [p - (1 - \varepsilon)e] / (R_d T), \quad \varepsilon = R_d/R_v = 0.622 \quad (17)$$

where e is the water-vapor partial pressure, obtained from relative humidity and the saturation vapor pressure. The rigorous CIPM-2007 formulation, recommended where accuracy better than 0.1 % is required, is [18]:

$$\rho = (p M_a / Z R T) [1 - x_v (1 - M_v/M_a)] \quad (18)$$

with Z the compressibility factor, x_v the vapour mole fraction, and M_a , M_v the molar masses of dry air and water. Table I gives the density reduction from dry to saturated conditions computed from (17). The effect reaches 2.8 % at 40 °C, which is of the same order as errors practitioners routinely attempt to correct by other means, yet it is absent from every standard atmosphere model in common use.

Table 1: Density reduction from dry to saturated air at 1013 hPa

Temperature (°C)	Saturation vapour pressure (hPa)	Density reduction, dry → saturated (%)
10	12.3	0.46
20	23.4	0.87
30	42.5	1.58
40	73.8	2.75

Computed from (17) with saturation vapor pressure from the Buck correlation; consistent with CIPM-2007 [18] to better than 0.02 %.

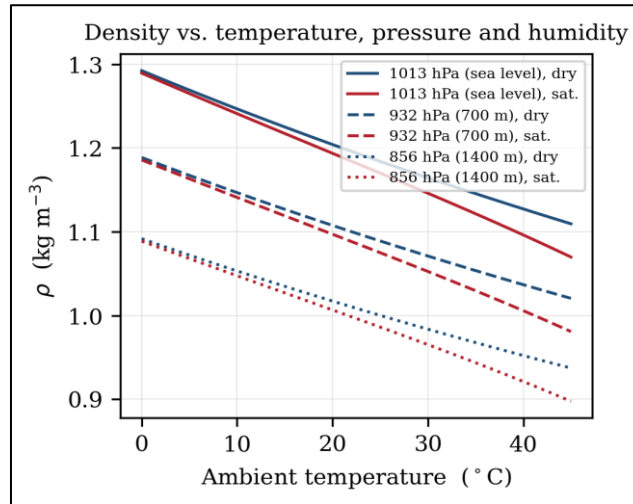


Figure 2: Air density as a function of ambient temperature at three station pressures, for dry and saturated air. The vertical separation between paired curves is the humidity contribution; the separation between line styles is the elevation contribution.

4.4 Altitude Traversed During Flight

Unlike the three preceding terms, this variation occurs within a single flight. Table II gives the density at apogee relative to the launch site for representative flight profiles, computed from (3).

Table 2: In-flight density variation by apogee class

Apogee (m AGL)	ρ at apogee vs. launch site	Adequacy of constant- ρ assumption
300	-3 %	Acceptable; error below C_D uncertainty
1 000	-9 %	Marginal; 2–4 % apogee error
3 000	-25 %	Invalid; profile integration required
10 000	-66 %	Invalid; full atmosphere model required

For A- through D-class model rockets reaching 100 to 400 m, treating ρ as constant at the launch-site value introduces errors below the uncertainty in C_D itself. For high-power certification flights exceeding 1 km, the altitude-dependent profile must be integrated.

4.5 Density Altitude as a Unified Metric

Aviation practice condenses all four effects into a single figure of merit: density altitude, defined as the ISA Geopotential altitude at which the standard density equals the observed local density [13], [22]. Inverting (3):

$$h_{da} = (T_0/L) \{ 1 - (\rho/\rho_0)^{1/4} [1 / (g/(L R_d) - 1)] \} \quad (19)$$

A launch conducted at 700 m elevation, 35 °C and 65 % relative humidity exhibits a density altitude of 1692 m — more than double the geometric elevation. This is a valuable field heuristic, since a single number replaces the full thermodynamic computation and can be communicated directly to a simulation package. Fig. 1 shows the corresponding offset-ISA profiles.

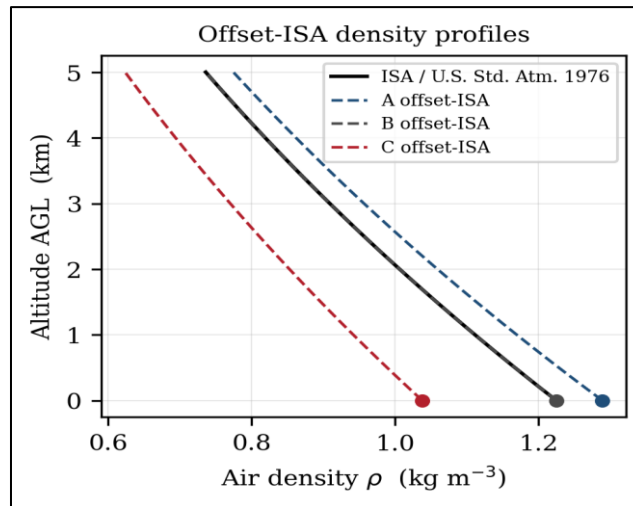


Figure 1: Density profiles for the three atmospheric states of the case study, each anchored to its measured surface density and propagated upward with the ISA lapse structure (offset-ISA). The solid line is the unmodified ISA. Markers denote measured launch-site density.

5 METHODOLOGY

5.1 Trajectory Model

A three-degree-of-freedom point-mass model was implemented, integrating (7) with a fixed-step scheme at $\Delta t = 2$ ms, verified against a step-halved solution to confirm step-size independence to better than 0.05 % in apogee. The model includes: time-varying mass from a prescribed burn rate; a thrust curve representative of a commercial ammonium-perchlorate composite motor with an initial pressure spike and mild regressively; a drag coefficient with a Reynolds-dependent skin-friction component after the Prandtl–Schlichting correlation [15] and a transonic compressibility rise above $M = 0.8$ [9], [24]; local speed of sound and dynamic viscosity evaluated from the instantaneous local temperature via Sutherland's law [17]; and the offset-ISA density profile of Section IV-E.

Pitch dynamics were evaluated post hoc from (13)–(15) along each trajectory using fixed mass-distribution and aerodynamic coefficients, an approximation valid for the small-disturbance analysis intended here.

5.2 Reference Vehicle

A representative single-stage high-power vehicle was defined; its parameters are given in Table III. The static margin of 3.7 calibers is deliberately conservative and typical of certification-flight practice [26].

Table 3: Reference Vehicle and Motor Parameters

Parameter	Symbol	Value
Body diameter	D	54 mm
Reference area	A_{ref}	$2.29 \times 10^{-3} \text{ m}^2$
Dry mass	m_d	0.780 kg
Propellant mass	m_p	0.0625 kg
Total impulse	I_t	160 N·s
Burn time	t_b	1.30 s
Baseline drag coefficient	C_{D0}	0.45
Total normal-force slope	$C_{N\alpha}$	14.5 rad^{-1}

Longitudinal inertia	I_L	$0.075 \text{ kg}\cdot\text{m}^2$
Static margin	—	3.7 cal

5.3 Atmospheric Cases

Three atmospheric states were defined to span the practical envelope:

- Case A cold, dry, sea level: 5 °C, 1030 hPa station pressure, 20 % relative humidity.
- Case B ISA reference: 15 °C, 1013.25 hPa, dry air. This is the baseline against which all changes are reported.
- Case C hot, humid, elevated site: 35 °C, 932 hPa station pressure (approximately 700 m elevation), 65 % relative humidity. This case is representative of afternoon conditions on the Nigerian high plains and comparable inland tropical plateaux.

Surface density for each case was computed from (17), and the vertical profile propagated with the ISA lapse structure. A supplementary sweep was run over launch-site densities from 0.90 to 1.35 kg m^{-3} to extract the empirical power-law exponent.

6 RESULTS AND DISCUSSION

Table IV summarizes the integrated results. Fig. 4 shows the altitude and velocity histories; Fig. 6 shows the density sweep.

Table 4: Case-Study Results, Referenced to Case B (ISA)

Quantity	A (cold)	B (ISA)	C (hot/humid/elevated)
Surface density ρ_s (kg m^{-3})	1.289	1.225	1.038
Density change vs. B (%)	+5.24	0.00	-15.26
Density altitude (m)	-535	0	1692
Apogee (m)	892	912	976
Apogee change (%)	-2.14	0.00	+7.10
Peak velocity (m s^{-1})	172.9	173.4	175.1
Peak velocity change (%)	-0.33	0.00	+0.98
Coast duration (s)	11.26	11.43	11.98
Coast duration change (%)	-1.51	0.00	+4.87
Peak ω_n (rad s^{-1})	41.0	40.1	37.3
ζ at burnout	0.0237	0.0231	0.0213

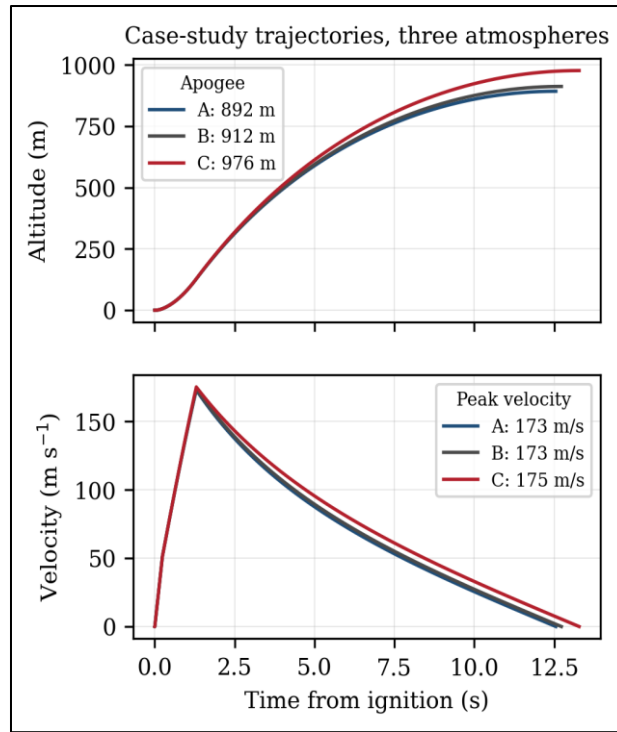


Figure 4: Altitude (upper) and velocity (lower) histories for the three atmospheric cases. The curves are visually close during the burn and diverge progressively through the coast, confirming that the coast phase is where density sensitivity is expressed.

6.1 Apogee

Apogee is by a wide margin the most density-sensitive metric. The 15.3 % density reduction of case C produces a 7.1 % apogee increase relative to the ISA baseline, an absolute difference of 65 m on a nominal 912 m flight. The full span between the cold and hot cases is 84 m, or 9.4 % of the cold-case apogee.

The density sweep of Fig. 6 yields an empirical power law $h \propto \rho^{-0.40}$ over the range examined. This exponent is consistent with the analytical prediction of (12): the vehicle operates at $V_b/V_t \approx 1.58$, for which (12) gives a coast-only sensitivity of approximately -0.55 . The numerically observed magnitude is smaller because a fraction of the total apogee is accumulated during the powered phase, where thrust rather than drag dominates the force balance and density sensitivity is correspondingly weak. Agreement between the closed-form and numerical results within this interpretation validates both.

The practical consequence is that the same airframe, flying the same motor, will reach materially different altitudes on different days. Competitive altitude events and certification attempts that do not report density altitude alongside apogee are reporting an incomplete result.

6.2 Peak Velocity

Peak velocity occurs at burnout in all three cases, since thereafter both gravity and drag decelerate the vehicle. The sensitivity is markedly weaker than for apogee: 0.98 % for a 15.3 % density reduction, corresponding to an empirical exponent of -0.058 . The physical reason is that during the burn the thrust term in (7) is roughly an order of magnitude larger than the drag term, so a change in the latter perturbs the result only weakly.

A secondary consequence deserves attention for faster vehicles. Where a design sits near $M = 0.8$, a low-density launch can push it across the transonic drag rise, and the compressibility penalty may partially or wholly offset the low-density advantage [9], [24]. Density-sensitivity is therefore not monotonic for transonic-capable designs, and simulation rather than extrapolation is required near this boundary.

6.3 Coast-Phase Duration

Coast duration increases by 4.87 % in case C relative to case B, an absolute increase of 550 ms. The sign follows directly from (10) and (11): lower density raises V_t , reduces the deceleration, and extends the time to apogee.

This is the most safety-relevant result in the paper. Commercial motor ejection delays are supplied in discrete increments, commonly 1 s apart, and are selected during pre-flight planning. A delay grain chosen for a cold, dense morning will fire before apogee on a hot, humid afternoon. Deployment at non-zero velocity produces high shock loads on the recovery harness, airframe “zippering”, and in severe cases parachute shredding or total recovery failure. Although the 550 ms shift computed here is smaller than a single delay increment, it compounds with motor delay-grain tolerance, which is itself of order $\pm 10\%$, and with thrust-curve lot variation. Delay selection should therefore be treated as a density-dependent decision.

The descent phase is affected in the opposite sense. Parachute terminal velocity scales as $\rho^{-1/2}$ by (11), so low-density conditions increase descent rate and, in combination with typically stronger afternoon convective winds, substantially enlarge the drift radius and recovery-area requirement [26].

6.4 Dynamic Stability and Centre-of-Pressure Behavior

This area is the most frequently misunderstood in amateur practice, and the distinction established in Section III-F is worth restating explicitly.

6.4.1 Static margin is density-invariant.

Because normal force and pitching moment both scale with $q = \frac{1}{2}\rho V^2$, the density term cancels identically in the centre-of-pressure calculation. The Barrowman result [3], [4] depends on geometry and, through compressibility, on Mach number — not on ρ . The static margin in calibers is therefore the same at sea level on a winter morning as at 2000 m density altitude in the tropics. Practitioners who verify stability solely by checking calibers of margin will observe no difference between the cases and may reasonably, but wrongly, conclude that the atmosphere is irrelevant to stability.

6.4.2 Dynamic response is strongly density-dependent.

From (13) and (15), $\omega_n \propto V\sqrt{\rho}$. The computed peak pitch natural frequency falls from 41.0 rad s^{-1} in case A to 37.3 rad s^{-1} in case C, a reduction of 9.0 %. The corresponding damping ratio at burnout falls from 0.0237 to 0.0213. Both values are very low in absolute terms, confirming the well-known result that slender passively stabilized rockets are lightly damped in pitch [23], [27]: a disturbance decays over many oscillation cycles rather than being suppressed within one. Lower density therefore means a slower corrective response, a larger angular excursion per unit gust impulse, and more cycles to settle. Fig. 5 shows the full histories.

6.4.3 Rail departure is the critical condition.

Minimum safe rail-exit velocity is conventionally specified as a fixed value, commonly 15 to 20 m s^{-1} [26]. The physically meaningful criterion, however, is a minimum dynamic pressure, since it is q that determines the corrective moment authority available to resist a crosswind at the instant the vehicle loses rail constraint. At the density of case C, a given velocity threshold corresponds to 15 % less dynamic pressure than at ISA. A rail length sized at sea level is therefore not conservative at a high-density-altitude site, and the velocity criterion should be scaled as $\rho^{-1/2}$.

6.4.4 Weathercocking is amplified.

The equilibrium angle of attack induced by a crosswind is set by the ratio of wind speed to vehicle speed, while the restoring moment that removes it scales with ρ . Low-density conditions therefore produce larger sustained weathercock angles and greater downrange displacement at apogee. Galejs's extension of the Barrowman treatment addresses precisely this wind-induced regime, in which the effective centre of pressure moves forward at finite angle of attack and the margin computed at zero incidence becomes optimistic [25].

6.4.5 Compressibility coupling.

At the higher speeds attainable in low-density air, Prandtl–Glauert compressibility increases the fin normal-force curve slope, shifting the centre of pressure aft and increasing static margin [9], [24]. This is a real but indirect and second-order density effect, mediated entirely through Mach number, and it acts in the stabilizing direction — partially offsetting the dynamic degradation described above.

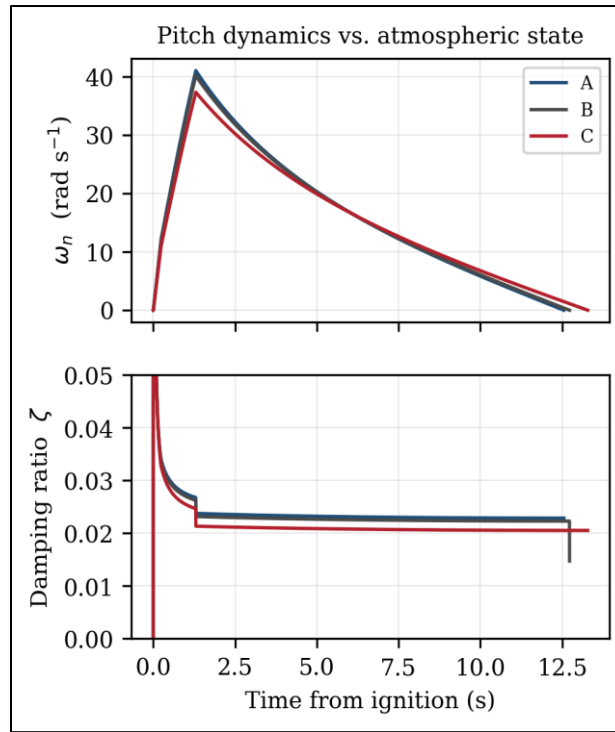


Figure 5: Pitch natural frequency (upper) and damping ratio (lower) for the three atmospheric cases, from ignition to apogee. Both quantities are reduced throughout the flight in the low-density case, indicating a slower and less well-damped response to disturbance.

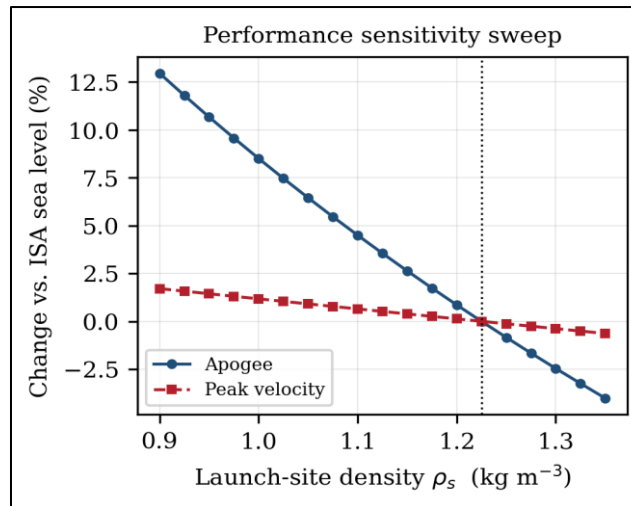


Figure 6: Parametric sweep of launch-site density. Apogee follows an empirical power law with exponent -0.40 ; peak velocity is an order of magnitude less sensitive.

7 ATMOSPHERIC AND SIMULATION MODELS

7.1 The International Standard Atmosphere

The ISA, standardized as ISO 2533 [12] and promulgated for aviation as ICAO Doc 7488 [13], defines a hydrostatic, dry, ideal-gas atmosphere with prescribed layer lapse rates. The closely related U.S. Standard Atmosphere 1976 [11] is identical below 32 km and extends the model to 1000 km. Its limitations for the present application are threefold and consequential:

The ISA is dry. Humidity is entirely absent from the model, so the effect quantified in Table I cannot be represented at all.

It represents a mid-latitude annual mean and is not a forecast of, or a fit to, any actual day at any actual site.

Its constant lapse rate is a smoothed idealization. Real boundary-layer profiles routinely exhibit superadiabatic surface layers and elevated inversions in precisely the 0 to 1 km band that model rocketry occupies [20].

The appropriate use of the ISA is therefore as a source of vertical structure, anchored to measured surface conditions — the offset-ISA approach used throughout this paper and illustrated in Fig. 1.

7.2 Higher-Fidelity and Empirical Models

For flights exceeding the troposphere or requiring statistical characterization, several alternatives exist. NRLMSISE-00 [21] is an empirical model of temperature and density from ground level to the exosphere, incorporating solar and geomagnetic activity indices, and is the standard reference for upper-atmosphere density. The NASA Earth Global Reference Atmospheric Model and the models catalogued in the AIAA guide [22] provide statistical dispersions as well as mean profiles. NASA-HDBK-1001 [20] documents the terrestrial environment criteria used for launch-vehicle design, including boundary-layer thermal structure. For serious altitude-record attempts, site-specific radiosonde ascents or numerical reanalysis data are superior to any generic model.

7.3 Simulation Packages

Three packages dominate current practice. OpenRocket [5] implements an extended Barrowman aerodynamic model with component-wise drag build-up over an ISA atmosphere; launch-site temperature and pressure are user-settable, and this is the mechanism by which density variation is handled, though humidity is not an available input. RASAero II offers higher-fidelity supersonic aerodynamic prediction and is preferable where the transonic coupling of Section VI-B is significant. The Cambridge Rocketry Simulator [6], [7] is distinguished by its stochastic six-degree-of-freedom formulation, propagating parameter uncertainty to a landing-zone probability distribution.

7.4 Recommended Density-Aware Workflow

On the basis of the foregoing, the following procedure is recommended for predictive rocketry:

- Measure launch-site temperature, station pressure and relative humidity immediately before flight. Station pressure, not QNH, is required.
- Compute density from (17), or from CIPM-2007 [18] where higher accuracy is warranted, and convert to density altitude via (19).
- Anchor the atmospheric profile to this surface density using the offset-ISA construction, rather than accepting default sea-level values.
- Integrate the full equations of motion with a Runge–Kutta or adaptive scheme, updating $C_D(M, Re)$ and $m(t)$ at every step.
- Run a Monte Carlo ensemble over the plausible ranges of ρ , C_D , total impulse and wind profile, and report apogee as a distribution with confidence bounds rather than a point estimate [6].
- Select the ejection delay against the low-density bound of that ensemble, since early deployment is the more damaging failure mode.

When validating against onboard barometric altimeter data, correct for the fact that such altimeters invert the ISA pressure relation internally [11]; in a non-standard atmosphere, both the trajectory and its measurement are in error, and the errors are correlated.

LIMITATIONS

The present analysis is subject to several limitations that should temper its quantitative conclusions. The trajectory model is three-degree-of-freedom and assumes vertical flight; it therefore cannot represent weathercocking displacement, and the dynamic-stability results of Section VI-D are obtained by post-hoc evaluation of the linearized coefficients along a vertical trajectory rather than by full angular integration. The drag model is semi-empirical and its absolute accuracy is limited by the assumed baseline C_{D0} ; however, since the study reports relative changes between atmospheric cases with C_{D0} held fixed, the conclusions are considerably more robust than the absolute apogee figures. The thrust curve is representative rather than measured from a specific certified motor. Finally, no flight-test validation is presented: the results are a computational study, and experimental confirmation across a deliberately wide range of measured density altitudes with an identical vehicle and motor lot remains the principal outstanding requirement.

RECOMMENDATIONS FOR FURTHER WORK

Controlled flight-test campaign: repeated flights of one airframe and one motor lot across a measured density-altitude range exceeding 1500 m, with onboard barometric and inertial instrumentation, to validate the exponent reported here.

Development of a density-altitude ejection-delay selection chart, analogous in function to Malewicky's altitude charts [1], to place delay selection on a quantitative footing.

Incorporation of relative humidity as a first-class input to open-source simulation packages, using the CIPM-2007 formulation [18].

Characterization of boundary-layer thermal structure at tropical and Sahelian launch sites, where surface superheating is strongest and where published profile data are sparse.

Extension to six-degree-of-freedom Monte Carlo analysis coupling density dispersion with wind-profile dispersion, to quantify the joint effect on landing-zone size.

Nomenclature

Symbol	Quantity	Unit
A_{ref}	Reference cross-sectional area	m^2
C_D	Total drag coefficient	—
$C_{N\alpha}$	Normal-force curve slope	rad^{-1}
C_1, C_2	Corrective and damping moment coefficients	$\text{N}\cdot\text{m}\cdot\text{rad}^{-1}, \text{N}\cdot\text{m}\cdot\text{s}\cdot\text{rad}^{-1}$
e	Water-vapour partial pressure	Pa
h_{da}	Density altitude	m
I_L	Longitudinal moment of inertia	$\text{kg}\cdot\text{m}^2$
k	Drag group, $\frac{1}{2}\rho C_D A_{\text{ref}}$	$\text{kg}\cdot\text{m}^{-1}$
L	Temperature lapse rate	$\text{K}\cdot\text{m}^{-1}$
M	Mach number	—
q	Dynamic pressure	Pa
R_d, R_v	Gas constant, dry air and water vapour	$\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$
Re	Reynolds number	—
V_b, V_t	Burnout and terminal velocity	$\text{m}\cdot\text{s}^{-1}$
$x_{\text{cp}}, x_{\text{cg}}$	Centre of pressure, centre of gravity	m
β	Velocity ratio squared, $(V_b/V_t)^2$	—
ε	Molar mass ratio, $R_d/R_v = 0.622$	—
ζ	Pitch damping ratio	—
μ	Dynamic viscosity	$\text{Pa}\cdot\text{s}$
ρ	Air density	$\text{kg}\cdot\text{m}^{-3}$
ω_n	Pitch natural frequency	$\text{rad}\cdot\text{s}^{-1}$

8 CONCLUSION

Atmospheric density influences model rocket performance principally through the drag term, where it acts linearly, and secondarily through the Reynolds number, where it acts weakly. A closed-form expression for coast-phase apogee sensitivity was derived and shown to be bounded between zero, for ballistic flight, and negative unity, for drag-limited flight, with the velocity ratio V_b/V_t as the governing parameter.

Numerical integration for a representative 54 mm high-power vehicle showed that a 15.3 % reduction in launch-site density — achievable through the combination of elevation, afternoon heating and high humidity encountered at inland tropical sites — increases apogee by 7.1 %, peak velocity by 1.0 % and coast duration by 4.9 %. Temperature and site elevation dominate the density budget; humidity contributes up to 2.8 % at high temperature and is conventionally, but unjustifiably, ignored.

The most consequential finding is the asymmetry in stability effects. Static margin is density-invariant, because density cancels from the centre-of-pressure computation, whereas pitch natural frequency and aerodynamic damping both scale with density and are degraded in low-density conditions. A practitioner who verifies stability by checking calibers of static margin alone will detect no difference between a dense and a rarefied atmosphere, while the vehicle's actual capacity to resist and recover from a gust at rail departure has been measurably reduced. Rail-exit velocity criteria should accordingly be scaled as $\rho^{-1/2}$ rather than treated as fixed.

Density-aware practice — measuring local conditions, computing true moist-air density, anchoring the atmospheric model to it, and propagating the resulting uncertainty through a Monte Carlo ensemble — requires only commodity instrumentation and represents the appropriate standard for both competitive altitude prediction and flight safety.

STATEMENTS ON COMPLIANCE WITH ETHICAL STANDARDS

Acknowledgments

I sincerely appreciate every member of the research team for your dedication, collaboration, and valuable contributions.

Disclosure of conflict of interest

There is no conflict of interest to be disclosed.

REFERENCES

- [1] D. J. Malewicki, "Model rocket altitude prediction charts including aerodynamic drag," Estes Industries, Penrose, CO, USA, Tech. Rep. TR-10, 1967.
- [2] G. M. Gregorek, "Aerodynamic drag of model rockets," Estes Industries, Penrose, CO, USA, Tech. Rep. TR-11, 1970.
- [3] J. S. Barrowman and J. A. Barrowman, "The theoretical prediction of the center of pressure," presented at the National Association of Rocketry Annual Meet (NARAM-8), Wooster, OH, USA, Aug. 1966.
- [4] J. S. Barrowman, "The practical calculation of the aerodynamic characteristics of slender finned vehicles," M.S. thesis, Dept. of Mech. Eng., The Catholic Univ. of America, Washington, DC, USA, 1967.
- [5] S. Niskanen, "Development of an open source model rocket simulation software," M.Sc. thesis, Dept. of Eng. Phys. and Math., Helsinki Univ. of Technology, Espoo, Finland, 2009.
- [6] S. Box, C. M. Bishop, and H. Hunt, "Stochastic six-degree-of-freedom flight simulator for passively controlled high-power rockets," *J. Aerosp. Eng.*, vol. 24, no. 1, pp. 31-45, Jan. 2011.
- [7] S. Box, C. M. Bishop, and H. Hunt, "Estimating the dynamic and aerodynamic parameters of passively controlled high power rockets for flight simulation," Dept. of Engineering, Univ. of Cambridge, Cambridge, U.K., Tech. Rep., 2009.
- [8] J. D. Anderson, *Fundamentals of Aerodynamics*, 6th ed. New York, NY, USA: McGraw-Hill Education, 2017.
- [9] J. D. Anderson, *Introduction to Flight*, 8th ed. New York, NY, USA: McGraw-Hill Education, 2016.
- [10] G. P. Sutton and O. Biblarz, *Rocket Propulsion Elements*, 9th ed. Hoboken, NJ, USA: John Wiley & Sons, 2017.
- [11] National Oceanic and Atmospheric Administration, National Aeronautics and Space Administration, and United States Air Force, *U.S. Standard Atmosphere, 1976*, NOAA-S/T 76-1562. Washington, DC, USA: U.S. Government Printing Office, 1976.
- [12] *Standard Atmosphere, ISO 2533:1975*, International Organization for Standardization, Geneva, Switzerland, 1975.
- [13] International Civil Aviation Organization, *Manual of the ICAO Standard Atmosphere: Extended to 80 Kilometres (262 500 Feet)*, 3rd ed., Doc 7488/3. Montreal, QC, Canada: ICAO, 1993.
- [14] S. F. Hoerner, *Fluid-Dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance*. Bricktown, NJ, USA: Hoerner Fluid Dynamics, 1965.
- [15] F. M. White, *Fluid Mechanics*, 7th ed. New York, NY, USA: McGraw-Hill, 2011.
- [16] Y. A. Cengel and M. A. Boles, *Thermodynamics: An Engineering Approach*, 8th ed. New York, NY, USA: McGraw-Hill Education, 2015.
- [17] P. T. Tsilingiris, "Thermophysical and transport properties of humid air at temperature range between 0 and 100 degrees C," *Energy Convers. Manage.*, vol. 49, no. 5, pp. 1098-1110, May 2008.

- [18] A. Picard, R. S. Davis, M. Glaser, and K. Fujii, "Revised formula for the density of moist air (CIPM-2007)," *Metrologia*, vol. 45, no. 2, pp. 149-155, Apr. 2008.
- [19] R. S. Davis, "Equation for the determination of the density of moist air (1981/91)," *Metrologia*, vol. 29, no. 1, pp. 67-70, Jan. 1992.
- [20] National Aeronautics and Space Administration, *Terrestrial Environment (Climatic) Criteria Handbook for Use in Aerospace Vehicle Development*, NASA-HDBK-1001. Washington, DC, USA: NASA, 2000.
- [21] J. M. Picone, A. E. Hedin, D. P. Drob, and A. C. Aikin, "NRLMSISE-00 empirical model of the atmosphere: Statistical comparisons and scientific issues," *J. Geophys. Res. Space Physics*, vol. 107, no. A12, pp. SIA 15-1-SIA 15-16, Dec. 2002.
- [22] *Guide to Reference and Standard Atmosphere Models*, ANSI/AIAA G-003C-2010, American Institute of Aeronautics and Astronautics, Reston, VA, USA, 2010.
- [23] R. C. Nelson, *Flight Stability and Automatic Control*, 2nd ed. New York, NY, USA: McGraw-Hill, 1998.
- [24] E. L. Fleeman, *Tactical Missile Design*, 2nd ed. Reston, VA, USA: American Institute of Aeronautics and Astronautics, 2006.
- [25] R. Galejs, "Wind instability: What Barrowman left out," *Sport Rocketry*, National Association of Rocketry, Marion, IA, USA, 1999.
- [26] National Association of Rocketry, *NAR Model Rocket Safety Code*. Marion, IA, USA: National Association of Rocketry, 2012.
- [27] M. Mandell, G. J. Caporaso, and W. P. Bengen, *Topics in Advanced Model Rocketry*. Cambridge, MA, USA: MIT Press, 1973.
- [28] ASHRAE, *ASHRAE Handbook - Fundamentals*, ch. 1: Psychrometrics. Atlanta, GA, USA: American Society of Heating, Refrigerating and Air-Conditioning Engineers, 2021.